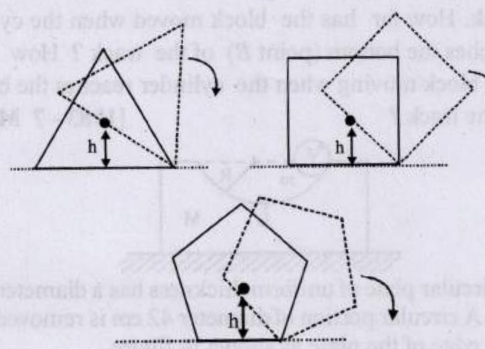
Chapter
System of Particles and Rotational Motion

Topic-1: Centre of Mass, Centre of Gravity & Principle of Moments

1 MCQs with One Correct Answer

1. Consider regular polygons with number of sides n = 3, 4, 5.... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as **[Adv. 2017]**



(a) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (b) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

(c) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (d) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

2. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is *m*. The mass of the ink used to draw the outer circle is 6 *m*.

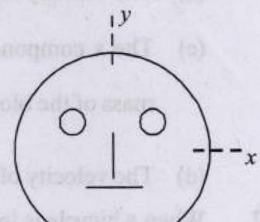
The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (– *a*, *a*), right inner circle (*a*, *a*), vertical line (0, 0) and horizontal line (0, – *a*). The *y*-coordinate of the centre of mass of the ink in this drawing is **[2009]**

(a) $\frac{a}{10}$

(b) $\frac{a}{8}$

(c) $\frac{a}{12}$

(d) $\frac{a}{3}$



3. Two particles *A* and *B* initially at rest, move towards each other under mutual force of attraction. At the instant when the speed of *A* is *V* and the speed of *B* is 2*V*, the speed of the centre of mass of the system is **(1982 - 3 Marks)**

(a) 3 *V* (b) *V*

(c) 1.5 *V* (d) zero

4 Fill in the Blanks

4. A rod of weight *w* is supported by two parallel knife edges *A* and *B* and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance *x* from *A* . The normal reaction on *A* is... and on *B* is..... **[1997 - 2 Marks]**

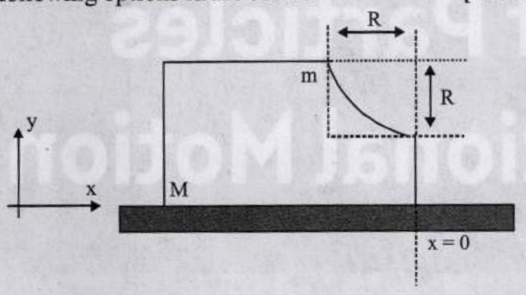
5 True / False

5. Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is 0.75 m/s. **[1989 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

6. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down.

When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct? **[Adv. 2017]**



(a) The position of the point mass m is: $x = -\sqrt{2}\dfrac{mR}{M+m}$

(b) The velocity of the point mass m is: $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(c) The x component of displacement of the center of mass of the block M is: $-\dfrac{mR}{M+m}$

(d) The velocity of the block M is: $V = -\dfrac{m}{M}\sqrt{2gR}$

7. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts **[1990 - 2 Marks]**

(a) in the backward direction on the front wheel and in the forward direction on the rear wheel.
(b) in the forward direction on the front wheel and in the backward direction on the rear wheel.
(c) in the backward direction on both the front and the rear wheels.
(d) in the forward direction on both the front the rear wheels.

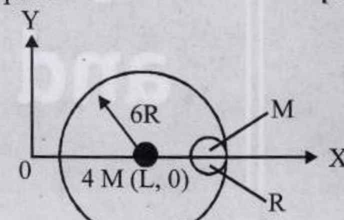
9 Assertion and Reason Type Questions

8. **STATEMENT-1:** If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant.
STATEMENT-2: The linear momentum of an isolated system remains constant. **[2007]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

10 Subjective Problems

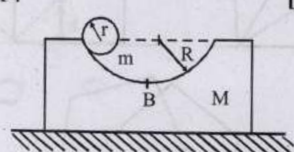
9. A small sphere of radius R is held against the inner surface of a larger sphere of radius $6R$ (Fig. P-3). The masses of large and small spheres are $4M$ and M, respectively, This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position. **[1996 - 3 Marks]**



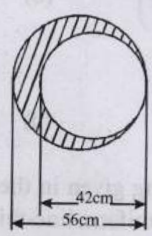
10. A uniform thin rod of mass M and length L is standing vertically along the y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive x-axis, and the rod starts falling. **[1993-1+5 Marks]**

(i) What is the path followed by the centre of mass of the rod during its fall?
(ii) Find the equation to the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point?

11. A block of mass M with a semicircular of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track ? **[1983 - 7 Marks]**



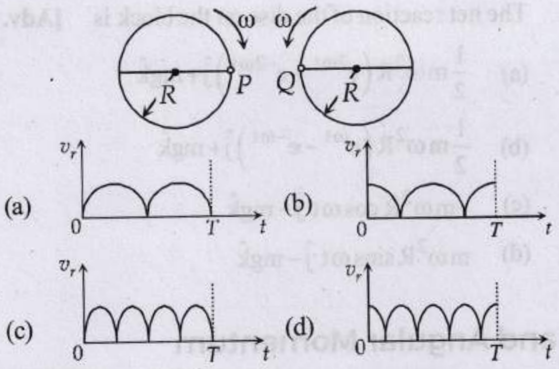
12. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in figure.
Find the position of the centre of mass of the remaining portion. **[1980]**



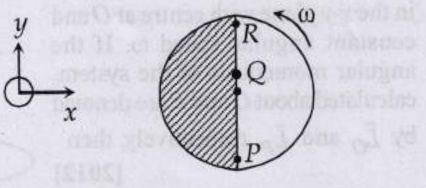
Topic-2: Angular Displacement, Velocity and Acceleration

1 MCQs with One Correct Answer

1. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **[2012]**



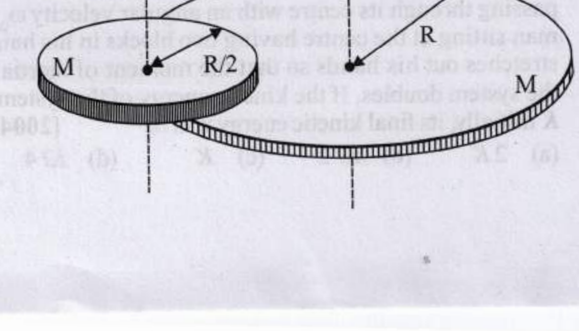
2. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **[2012]**



(a) P lands in the shaded region and Q in the unshaded region.
(b) P lands in the unshaded region and Q in the shaded region.
(c) Both P and Q land in the unshaded region.
(d) Both P and Q land in the shaded region.

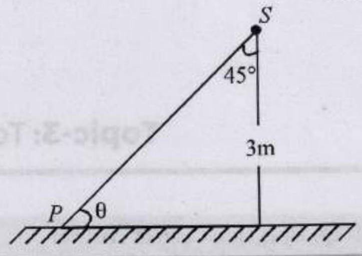
3 Numeric Answer

3. A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass M and radius $\frac{R}{2}$ is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω. If the angular speed at which the large disc rotates is $\frac{\omega}{n}$, then the value of n is ______. **[Adv. 2024]**



4 Fill in the Blanks

4. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 radian/second. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when $\theta = 45°$ (see fig.) is .... m/s **[1987 - 2 Marks]**



8 Comprehension Based Questions

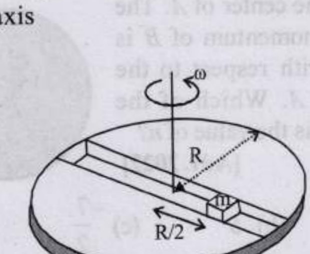
Passage

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{v}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega}.$$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis



perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega \hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r}(R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.

5. The distance r of the block at time t is **[Adv. 2016]**

(a) $\frac{R}{4}\left(e^{\omega t}+e^{-\omega t}\right)$ (b) $\frac{R}{2}\cos\omega t$

(c) $\frac{R}{4}\left(e^{2\omega t}+e^{-2\omega t}\right)$ (d) $\frac{R}{2}\cos 2\omega t$

6. The net reaction of the disc on the block is **[Adv. 2016]**

(a) $\frac{1}{2}m\omega^2R\left(e^{2\omega t}-e^{-2\omega t}\right)\hat{j}+mg\hat{k}$

(b) $\frac{1}{2}m\omega^2R\left(e^{\omega t}-e^{-\omega t}\right)\hat{j}+mg\hat{k}$

(c) $-m\omega^2R\cos\omega t\,\hat{j}-mg\hat{k}$

(d) $m\omega^2R\,\text{sins}\,\omega t\,\hat{j}-mg\hat{k}$

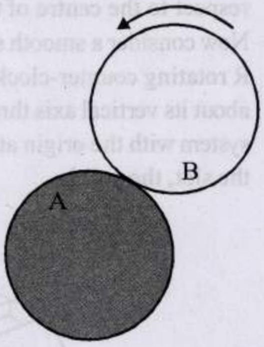
Topic-3: Torque, Couple and Angular Momentum

1 MCQs with One Correct Answer

1. A bar of mass M = 1.00 kg and length L = 0.20 m is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass m = 0.10 kg is moving on the same horizontal surface with 5.00 m s^{-1} speed on a path perpendicular to the bar. It hits the bar at a distance L/2 from the pivoted end and returns back on the same path with speed v. After this elastic collision, the bar rotates with an angular velocity ω. Which of the following statement is correct? **[Adv. 2023]**

(a) ω = 6.98 rad s^{-1} and v = 4.30 m s^{-1}
(b) ω = 3.75 rad s^{-1} and v = 4.30 m s^{-1}
(c) ω = 3.75 rad s^{-1} and v = 10.0 m s^{-1}
(d) ω = 6.80 rad s^{-1} and v = 4.10 m s^{-1}

2. A flat surface of a thin uniform disk *A* of radius *R* is glued to a horizontal table. Another thin uniform disk *B* of mass *M* and with the same radius *R* rolls without slipping on the circumference of A, as shown in the figure. A flat surface of *B* also lies on the plane of the table. The center of mass of *B* has fixed angular speed ω about the vertical axis passing through the center of *A*. The angular momentum of *B* is $nM\omega R^2$ with respect to the center of *A*. Which of the following is the value of *n*? **[Adv. 2022]**



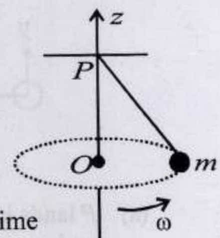
(a) 2 (b) 5 (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

3. A uniform wooden stick of mass 1.6 kg and length *l* rests in an inclined manner on a smooth, vertical wall of height h(< *l*) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/*l* and the frictional force *f* at the bottom of the stick are (g = 10 m s^{-2}) **[Adv. 2016]**

(a) $\frac{h}{l}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}$ N (b) $\frac{h}{l}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}$ N

(c) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}$ N (d) $\frac{h}{l}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}$ N

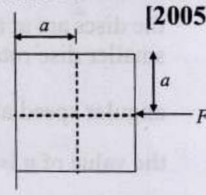
4. A small mass *m* is attached to a massless string whose other end is fixed at *P* as shown in the figure. The mass is undergoing circular motion in the *x-y* plane with centre at *O* and constant angular speed ω. If the angular momentum of the system, calculated about *O* and *P* are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then **[2012]**



(a) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time
(b) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant
(c) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time
(d) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

5. A particle is confined to rotate in a circular path decreasing linear speed, then which of the following is correct? **[2005S]**
(a) $\vec{L}$ (angular momentum) is conserved about the centre
(b) only direction of angular momentum $\vec{L}$ is conserved
(c) It spirals towards the centre
(d) its acceleration is towards the centre.

6. A block of mass *m* is at rest under the action of force *F* against a wall as shown in figure. Which of the following statement is incorrect? **[2005S]**

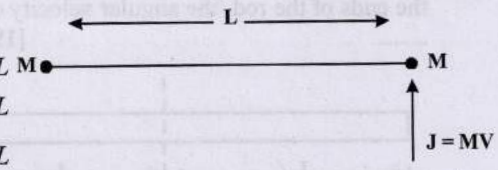


(a) *f* = mg [*f* friction force]
(b) *F* = *N* [*N* normal force]
(c) *F* will not produce torque
(d) *N* will not produce torque

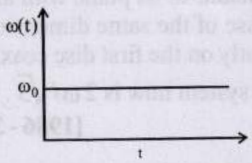
7. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_o. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is *K* initially, its final kinetic energy will be **[2004S]**
(a) 2*K* (b) *K*/2 (c) *K* (d) *K*/4

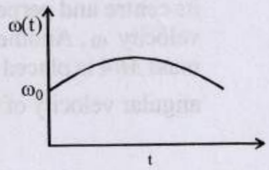
8. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved? **[2003S]**
(a) centre of the circle
(b) on the circumference of the circle.
(c) inside the circle
(d) outside the circle.

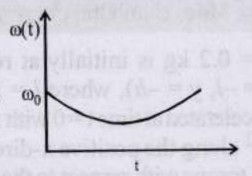
9. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = MV$ is imparted to the body at one of its ends, what would be its angular velocity? **[2003S]**
(a) V/L
(b) $2V/L$
(c) $V/3L$
(d) $V/4L$

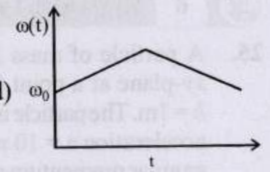


10. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as **[2002S]**

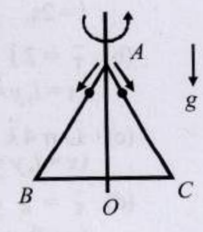
(a) 

(b) 

(c) 

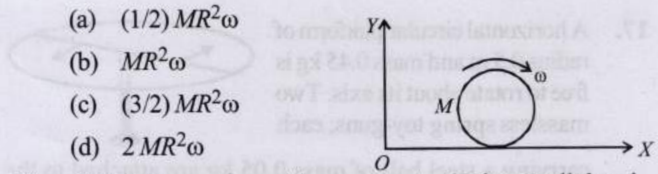
(d) 

11. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are **[2000S]**



(a) angular velocity and total energy (kinetic and potential)
(b) Total angular momentum and total energy
(c) angular velocity and moment of inertia about the axis of rotation
(d) total angular momentum and moment of inertia about the axis of rotation

12. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Figure. The magnitude of angular momentum of the disc about the origin O is **[1999S - 2 Marks]**
(a) $(1/2)MR^2\omega$
(b) $MR^2\omega$
(c) $(3/2)MR^2\omega$
(d) $2MR^2\omega$



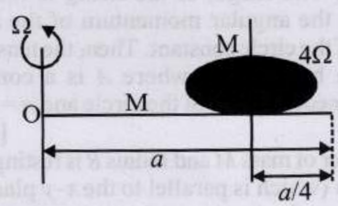
13. A mass M moving with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin **[1985 - 2 Marks]**
(a) is zero
(b) remains constant
(c) goes on increasing
(d) goes on decreasing

14. A thin circular ring of mass 'M and radius r is rotating about its axis with a constant angular velocity ω, Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity **[1983 - 1 Mark]**
(a) $\dfrac{\omega M}{(M+m)}$
(b) $\dfrac{\omega(M-2m)}{(M+2m)}$
(c) $\dfrac{\omega M}{(M+2m)}$
(d) $\dfrac{\omega(M+2m)}{M}$

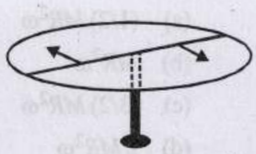
2 Integer/Non-negative Integer Value Answer

15. A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j})\,kg\ ms^{-2}$ with $k = 1\ kg\ s^{-2}$. At time $t = 0$, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right) m$ and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right) ms^{-1}$. Let v_x and v_y denote the x and the y components of the particle's velocity, respectively. **Ignore gravity.** When $z = 0.5\ m$, the value of $(xv_y - yv_x)$ is ______ m^2s^{-1}. **[Adv. 2022]**

16. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius a/4 is pivoted on this rod with its center at a distance a/4 from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)$n. The value of n is ______. **[Adv. 2021]**



17. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0 25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **[Adv. 2014]**



18. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[Adv. 2013]**

19. A binary star consists of two stars A (mass $2.2M_s$) and B (mass $11M_s$), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is **[2010]**

3 Numeric Answer

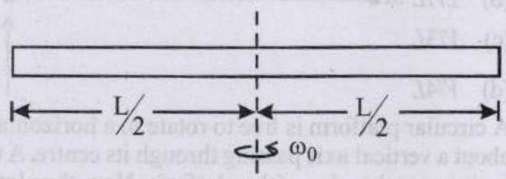
20. Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the center (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is ______. **[Adv. 2020]**

4 Fill in the Blanks

21. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle. (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$ where A is a constant, r is the instantaneous radius of the circle and n = ... **[1993 - 1 Mark]**

22. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x–y plane) with its axis fixed along the y-axis and free to rotate about its axis. The platform is given a motion in the x-direction given by $x = A\cos(\omega t)$. There is no slipping between the cylinder and platform. The maximum torque acting on the cylinder during its motion is .... **[1988 - 2 Marks]**

23. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpenducular to the rod and passing through the midpoint of the rod (see figure). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is .... **[1988 - 2 Marks]**



5 True / False

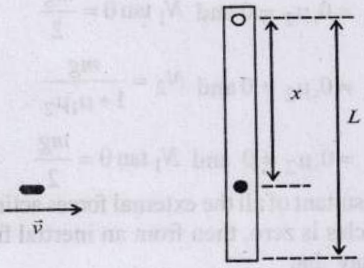
24. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{5}$. **[1986 - 3 Marks]**

6 MCQs with One or More than One Correct Answer

25. A particle of mass M = 0.2 kg is initially at rest in the xy-plane at a point ($x = -l$, $y = -h$), where l = 10 m and h = 1m. The particle is accelerated at time t = 0 with a constant acceleration a = 10 m/s^2 along the positive x-direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$, respectively. $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z-directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct? **[Adv. 2021]**
   (a) The particle arrives at the point ($x = l$, $y = -h$) at time t = 2s.
   (b) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point ($x = l$, $y = -h$)
   (c) $\vec{L} = 4\hat{k}$ when the particle passes through the point ($x = l$, $y = -h$)
   (d) $\vec{\tau} = \hat{k}$ when the particle passes through the point ($x = 0$, $y = -h$)

26. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot.

The maximum angular speed ω_M is achieved for $x = x_M$. Then **[Adv. 2020]**



(a) $\omega = \dfrac{3vx}{L^2 + 3x^2}$ (b) $\omega = \dfrac{12vx}{L^2 + 12x^2}$

(c) $x_M = \dfrac{L}{\sqrt{3}}$ (d) $\omega_M = \dfrac{v}{2L}\sqrt{3}$

27. Consider a body of mass 1.0 kg at rest at the origin at time $t = 0$. A force $\vec{F} = (\alpha t\,\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ Ns^{-1}$ and $\beta = 1.0\ N$. The torque acting on the body about the origin at time $t = 1.0\ s$ is $\vec{\tau}$. Which of the following statements is (are) true? **[Adv. 2018]**

(a) $|\vec{\tau}| = \frac{1}{3}\ Nm$

(b) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(c) The velocity of the body at $t = 1\ s$ is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\ ms^{-1}$

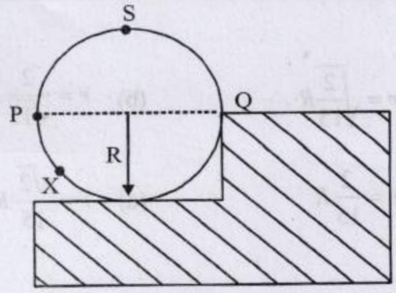
(d) The magnitude of displacement of the body at $t = 1\ s$ is $\frac{1}{6}\ m$

28. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true? **[Adv. 2018]**

(a) $v = \sqrt{\dfrac{k}{2m}}R$ (b) $v = \sqrt{\dfrac{k}{m}}R$

(c) $L = \sqrt{mk}R^2$ (d) $L = \sqrt{\dfrac{mk}{2}}R^2$

29. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct? **[Adv. 2017]**



(a) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

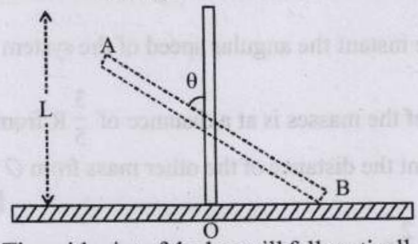
(b) If the force is applied normal to the circumference at point X then τ is constant

(c) If the force is applied normal to the circumference at point P then τ is zero

(d) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

30. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure).

At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct? **[Adv. 2017]**



(a) The midpoint of the bar will fall vertically downward

(b) The trajectory of the point A is a parabola

(c) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(d) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

31. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j},$$

where $\alpha = 10/3\ ms^{-3}$, $\beta = 5\ ms^{-2}$ and $m = 0.1$ kg. At $t = 1$ s, which of the following statement(s) is(are) true about the particle? **[Adv. 2016]**

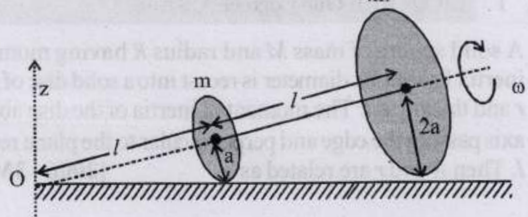
(a) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\ ms^{-1}$

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -5/3)\ \hat{k}$ N m s

(c) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\ \hat{k}$ N m

32. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centres. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is (are) true? **[Adv. 2016]**



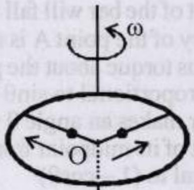
(a) The centre of mass of the assembly rotates about the z-axis with an angular speed of $\omega/5$

(b) The magnitude of angular momentum of center of mass of the assembly about the point O is $81 ma^2\omega$

(c) The magnitude of angular momentum of the assembly about its center of mass is $17 ma^2\omega/2$.

(d) The magnitude of the z-component of $\vec{L}$ is $55 ma^2\omega$.

33. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$R from O. At this instant the distance of the other mass from O is **[Adv. 2015]**



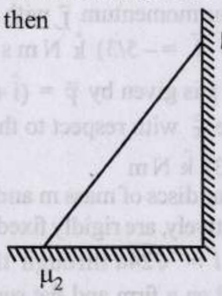
(a) $\frac{2}{3}$R

(b) $\frac{1}{3}$R

(c) $\frac{3}{5}$R

(d) $\frac{4}{5}$R

34. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then **[Adv. 2014]**



(a) $\mu_1 = 0, \mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(b) $\mu_1 \neq 0, \mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(c) $\mu_1 \neq 0, \mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(d) $\mu_1 = 0, \mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

35. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[2009]**

(a) linear momentum of the system does not change in time

(b) kinetic energy of the system does not change in time

(c) angular momentum of the system does not change in time

(d) potential energy of the system does not change in time

36. The torque τ on a body about a given point is found to be equal to $A \times L$ where A is a constant vector, and L is the angular momentum of the body about that point. From this it follows that **[1998S - 2 Marks]**

(a) $\frac{dL}{dt}$ is perpendicular to L at all instants of time.

(b) the component of L in the direction of A does not change with time.

(c) the magnitude of L does not change with time.

(d) L does not change with time

37. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **[1990 - 2 Marks]**

(a) zero (b) $\frac{mv^3}{4\sqrt{2}g}$ (c) $\frac{mv^3}{\sqrt{2}g}$ (d) $m\sqrt{2gh^3}$

10 Subjective Problems

38. A particle is projected at time $t = 0$ from a point P on the ground with a speed v_0, at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t = v_0/g$ **[1984- 6 Marks]**

Topic-4: Moment of Inertia and Rotational K.E.

1 MCQs with One Correct Answer

1. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. Then R and r are related as **[2006 - 3M, –1]**

(a) $r = \sqrt{\frac{2}{15}}R$

(b) $r = \frac{2}{\sqrt{15}}R$

(c) $r = \frac{2}{15}R$

(d) $r = \frac{\sqrt{2}}{15}R$

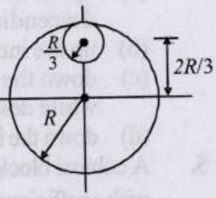
2. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The m oment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is **[2005S]**

(a) $4MR^2$

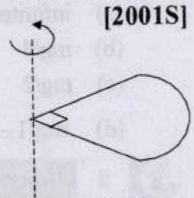
(b) $\frac{40}{9}MR^2$

(c) $10\ MR^2$

(d) $\frac{37}{9}MR^2$



3. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is **[2001S]**



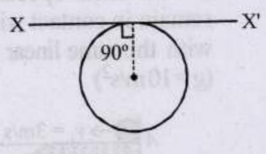
(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$

(c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}\ MR^2$

4. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is **[2000S]**

(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$



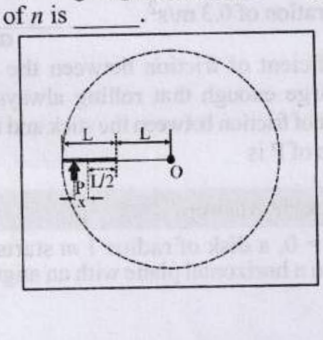
5. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to **[1998S - 2 Marks]**

(a) I (b) $I \sin^2 \theta$

(c) $I \cos^2 \theta$ (d) $I \cos^2 (\theta/2)$

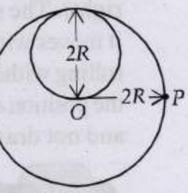
2 Integer Value Answer

6. A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance $x = \frac{L}{n}$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is ______. **[Adv. 2024]**



7. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is **[Adv. 2015]**

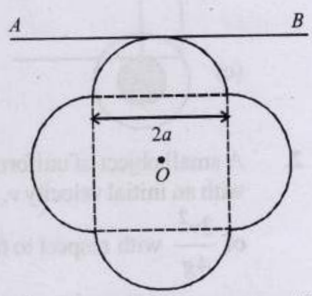
8. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P/I_O to the nearest integer is **[2012]**



9. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg– m², then N is. **[2011]**

4 Fill in the Blanks

10. A symmetric lamina of mass M consists of a square shape with a semicircular section over of the edge of the square as shown in Fig. P-10. The side of the square is $2a$. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is $1.6\ Ma^2$.



The moment of inertia of the lamina about the tangent AB in the plane of the lamina is.... **[1997 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

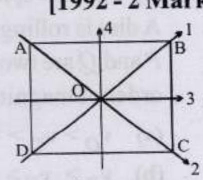
11. The moment of inertia of a thin square plate $ABCD$, Fig., of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is **[1992 - 2 Marks]**

(a) $I_1 + I_2$

(b) $I_3 + I_4$

(c) $I_1 + I_3$

(d) $I_1 + I_2 + I_3 + I_4$

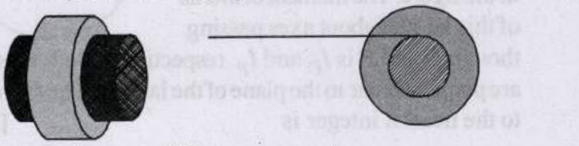


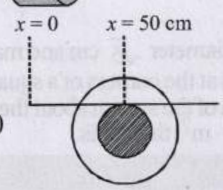
where I_1, I_2, I_3 and I_4 are respectively the moments of intertial about axis 1, 2, 3 and 4 which are in the plane of the plate.

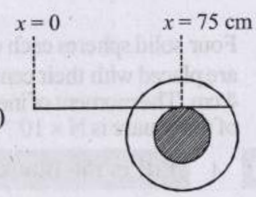
Topic-5: Rolling Motion

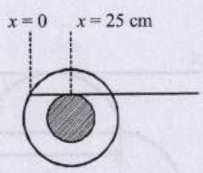
1 MCQs with One Correct Answer

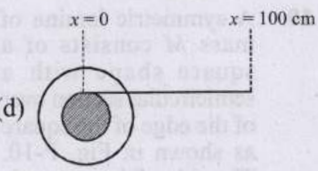
1. A small roller of diameter 20 cm has an axle of diameter 10 cm (see figure below on the left). It is on a horizontal floor and a meter scale is positioned horizontally on its axle with one edge of the scale on top of the axle (see figure on the right). The scale is now pushed slowly on the axle so that it moves without slipping on the axle, and the roller starts rolling without slipping. After the roller has moved 50 cm, the position of the scale will look like (figures are schematic and not drawn to scale) **[Adv. 2020]**



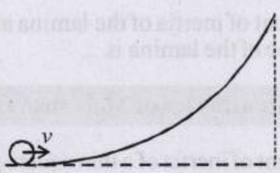
(a)


(b)


(c)


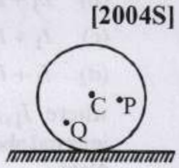
(d)


2. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is **[2007]**



(a) ring (b) solid sphere
(c) hollow sphere (d) disc

3. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is **[2004S]**

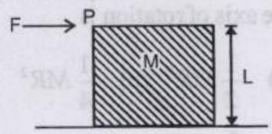


(a) $v_Q > v_C > v_P$
(b) $v_P > v_C > v_Q$
(c) $v_P = v_C, v_Q = v_C/2$
(d) $v_P < v_C > v_Q$

4. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are **[2002S]**

(a) up the incline while ascending and down the incline descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending.

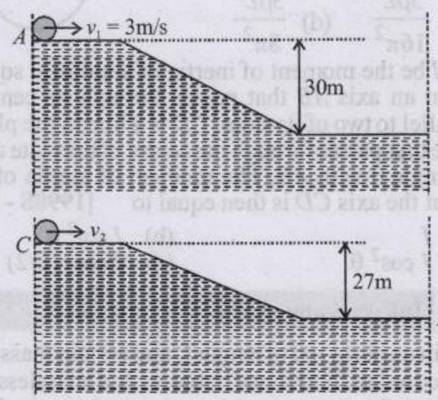
5. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is **[2000S]**



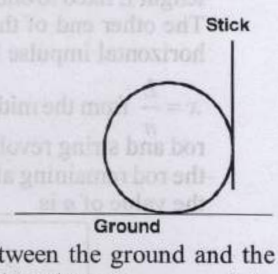
(a) infinitesimal
(b) mg/4
(c) mg/2
(d) mg(1 − μ)

2 Integer Value Answer

6. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s then v_2 in m/s is ($g = 10\,m/s^2$) **[Adv. 2015]**



7. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2.

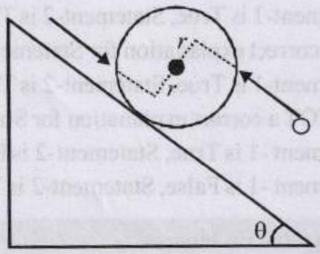


The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is **[2011]**

3 Numeric Answer

8. At time $t = 0$, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha = \frac{2}{3} rad\, s^{-2}$. A small stone is stuck to the disk. At $t = 0$, it is at the contact point of the disk and the plane. Later, at time $t = \sqrt{\pi} s$, the stone detaches itself and flies off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is $\frac{1}{2} + \frac{x}{10}$. The value of x is ______. [Take $g = 10\, ms^{-2}$.] **[Adv. 2022]**

9. A solid sphere of mass 1 kg and radius 1 m rolls without slipping on a fixed inclined plane with an angle of inclination $\theta = 30°$ from the horizontal. Two forces of magnitude 1 N each, parallel to the incline, act on the sphere, both at distance $r = 0.5\ m$ from the center of the sphere, as shown in the figure. The acceleration of the sphere down the plane is ______ ms^{-2}. (Take $g = 10\ ms^{-2}$.) **[Adv. 2022]**



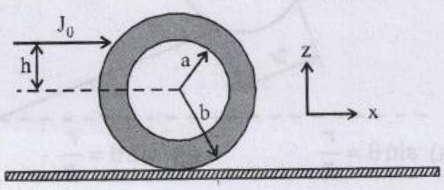
10. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is ______. Take $g = 10\ \text{ms}^{-2}$. **[Adv. 2018]**

5 True / False

11. A ring of mass 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius are given the same kinetic energy and released simultaneously on a flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from the ring and the cylinder. The rolling friction in both cases is negligible. The cylinder will reach the wall first. **[1989 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

12. An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $j_0 \hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct?



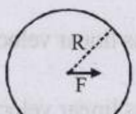
(a) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$. **[Adv. 2023]**

(b) For $\mu \neq 0$ and $a \to 0$, $h_m = b$.

(c) For $h = h_m$ the initial angular velocity does not depend on the inner radius a.

(d) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

13. A horizontal force F is applied at the center of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is m. The center of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct? **[Adv. 2021]**



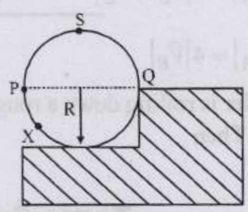
(a) For the same F, the value of a does not depend on whether the cylinder is solid or hollow

(b) For a solid cylinder, the maximum possible value of a is $2\mu g$

(c) The magnitude of the frictional force on the object due to the ground is always μmg

(d) For a thin-walled hollow cylinder, $a = \frac{F}{2m}$

14. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct? **[Adv. 2017]**



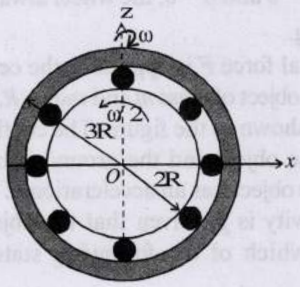
(a) If the force is applied at point P tangentially then decreases continuously as the wheel climbs

(b) If the force is applied normal to the circumference at point X then τ is constant

(c) If the force is applied normal to the circumference at point P then τ is zero

(d) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

15. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearings. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, **[2012]**



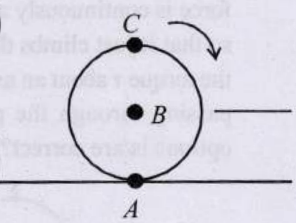
(a) the point O has linear velocity $3R\omega\hat{i}$

(b) the point P has linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$.

(c) the point P has linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(d) the point P has linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

16. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then, **[2009]**



(a) $\vec{V}_C-\vec{V}_A=2(\vec{V}_B-\vec{V}_C)$

(b) $\vec{V}_C-\vec{V}_B=\vec{V}_B-\vec{V}_A$

(c) $|\vec{V}_C-\vec{V}_A|=2|\vec{V}_B-\vec{V}_C|$

(d) $|\vec{V}_C-\vec{V}_A|=4|\vec{V}_B|$

17. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then **[2006 - 5M, –1]**

(a) The friction force is dissipative

(b) The friction force is necessarily changing

(c) The friction force will aid rotation but hinder translation

(d) The friction force is reduced if θ is reduced

9 Assertion and Reason Type Questions

18. **STATEMENT-1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

STATEMENT-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline. **[2008]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

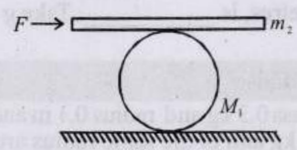
(c) Statement -1 is True, Statement-2 is False

(d) Statement -1 is False, Statement-2 is True

10 Subjective Problems

19. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown in Figure. There in no slipping at any contact. The horizontal component of the force applied by the man is F. **[1999 - 10 Marks]**

Find



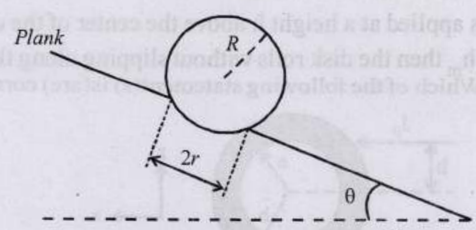
(a) the accelerations of the plank and the center of mass of the cylinder, and

(b) the magnitudes and directions of frictional forces at contact points.

Topic-6: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

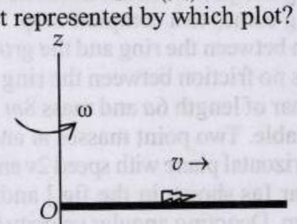
1. A football of radius R is kept on a hole of radius r ($r < R$) made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in the figure below. The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale) **[Adv. 2020]**

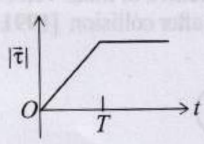


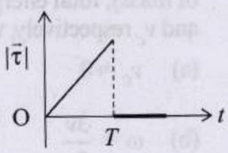
(a) $\sin\theta=\frac{r}{R}$ (b) $\tan\theta=\frac{r}{R}$

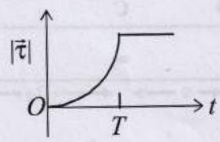
(c) $\sin\theta=\frac{r}{2R}$ (d) $\cos\theta=\frac{r}{2R}$

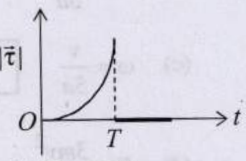
2. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v, with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) about O, as a function of time is best represented by which plot? **[2012]**



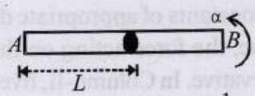
(a)


(b)


(c)


(d)


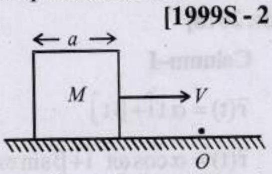
3. A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is **[2000S]**



(a) $\sqrt{\mu/\alpha}$ (b) $\mu/\sqrt{\alpha}$ (c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

4. A cubical block of side a is moving with velocity V on a horizontal smooth plane as shown in Figure. It hits a ridge at point O. The angular speed of the block after it hits O is **[1999S - 2 Marks]**

(a) $3V/(4a)$
(b) $3V/(2a)$
(c) $\sqrt{3V}/(\sqrt{2a})$
(d) zero



5. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity υ. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B, respectively. Then **[1999S - 2 Marks]**

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$
(c) $\omega_A = \omega$ (d) $\omega_B = \omega$

6. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of **[1995S]**

(a) 0.42 m from mass of 0.3 kg
(b) 0.70 m from mass of 0.7 kg
(c) 0.98 m from mass of 0.3 kg
(d) 0.98 m from mass of 0.7 kg

7. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A pendulum bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with track is **[1992 - 2 Mark]**

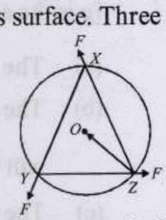
(a) zero (b) 30° (c) 45° (d) 60°

8. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is **[1992 - 2 Marks]**

(a) $\frac{M\omega^2 L}{2}$ (b) $M\omega^2 L$
(c) $\frac{M\omega^2 L}{4}$ (d) $\frac{M\omega^2 L^2}{2}$

2 Integer Value Answer

9. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure).



One second after applying the forces, the angular speed of the disc in rad s^{-1} is **[Adv. 2014]**

3 Numeric Value/New Stem Base Questions

Stem for Q. No. 10-11

A pendulum consists of a bob of mass $m = 0.1$ kg and a massless inextensible string of length $L = 1.0$ m. It is suspended from a fixed point at height $H = 0.9$ m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse $P = 0.2$ kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg-m²/s. The kinetic energy of the pendulum just after the lift-off is K Joules.

10. The value of J is ____ **[Adv. 2021]**
11. The value of K is ____ **[Adv. 2021]**

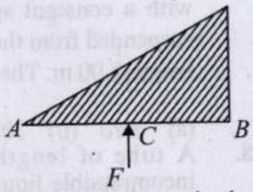
4 Fill in the Blanks

12. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre

of the face, at a height $3a/4$ above the base. The minimum value of F for which the cube begins to tip about the edge is .... (Assume that the cube does not slide). **[1984 - 2 Marks]**

5 True / False

13. A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and (a) passing through A, (b) passing through B, by the application of the same force, F, at C (midpoint of AB) as shown in the figure. The angular acceleration in both the cases will be the same. **[1985 - 3 Marks]**



6 MCQs with One or More than One Correct Answer

14. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ? **[Adv. 2019]**
[g is the acceleration due to gravity]

(a) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(b) The radical acceleration of the rod's center of mass will be $\frac{3g}{4}$

(c) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

(d) The angular acceleration of the rod will be $\frac{2g}{L}$

15. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? **[2012]**

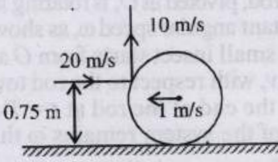
(a) Both cylinders P and Q reach the ground at the same time.

(b) Cylinders P has larger linear acceleration than cylinder Q.

(c) Both cylinders reach the ground with same translational kinetic energy.

(d) Cylinder Q reaches the ground with larger angular speed.

16. A thin ring of mass 2 kg and radius 0.5 m is rolling without on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[2011]**



(a) the ring has pure rotation about its stationary CM.

(b) the ring comes to a complete stop.

(c) friction between the ring and the ground is to the left.

(d) there is no friction between the ring and the ground.

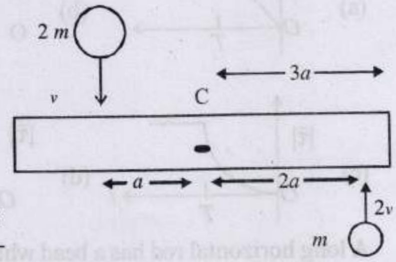
17. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speed $2v$ and v, respectively, strike the bar [as shown in the fig.] and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_c respectively, we have after collision **[1991 - 2 Mark]**

(a) $v_c = 0$

(b) $\omega = \frac{3v}{5a}$

(c) $\omega = \frac{v}{5a}$

(d) $E = \frac{3mv^2}{5}$



7 Match the Following

18. In the Column-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$ In each case, the force acting on the particle is either zero or conservative. In Column-II, five physical quantities of the particle are mentioned $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path. **[Adv. 2018]**

	Column–I		Column–II
P.	$\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$	1.	$\vec{p}$
Q.	$\vec{r}(t) = \alpha\cos\omega t\ \hat{i} + \beta\sin\omega t\ \hat{j}$	2.	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos\omega t\ \hat{i} + \sin\omega t\ \hat{j})$	3.	K
S.	$\vec{r}(t) = \alpha t\ \hat{i} + \frac{\beta}{2}t^2\ \hat{j}$	4.	U
		5.	E

(a) P→1, 2, 3, 4, 5; Q→2, 5; R→2, 3, 4, 5; S→5

(b) P→1, 2, 3, 4, 5; Q→3, 5; R→2, 3, 4, 5; S→2, 5

(c) P→2, 3, 4; Q→5; R→1, 2, 4; S→2, 5

(d) P→1, 2, 3, 5; Q→2, 5; R→2, 3, 4, 5; S→2, 5

19. **Column-II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column-I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II**. **[2009]**

Column-I

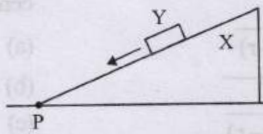
(A) The force exerted by X on Y has a magnitude Mg.

(B) The gravitational potential energy of X is continuously increasing.

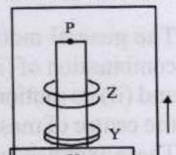
(C) Mechanical energy of the system X + Y is continuously decreasing.

(D) The torque of the weight of Y about point P is zero.

Column II

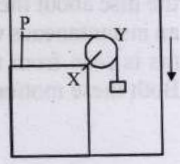
(p) 

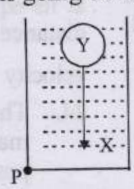
Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.

(q) 

Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

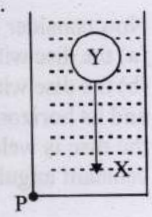
(r) A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.



(s) 

A sphere Y of mass M is put in a non-viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

(t) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.



8 Comprehension Based Questions

Passage 1

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g. **[Adv. 2017]**

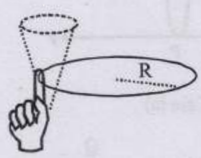


Figure 1

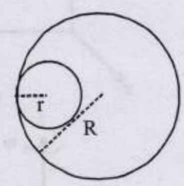


Figure 2

20. The total kinetic energy of the ring is

(a) $M\omega_0^2R^2$
(b) $\frac{1}{2}M\omega_0^2(R-r)^2$
(c) $M\omega_0^2(R-r)^2$
(d) $\frac{3}{2}M\omega_0^2(R-r)^2$

21. The minimum value of ω_0 below which the ring will drop down is

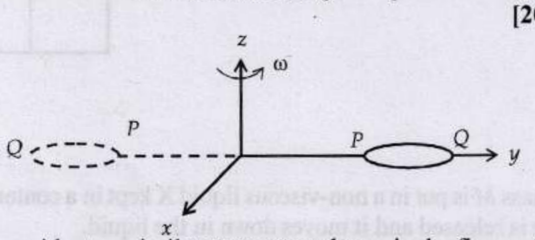
(a) $\sqrt{\dfrac{g}{\mu(R-r)}}$ (b) $\sqrt{\dfrac{2g}{\mu(R-r)}}$

(c) $\sqrt{\dfrac{3g}{2\mu(R-r)}}$ (d) $\sqrt{\dfrac{g}{2\mu(R-r)}}$

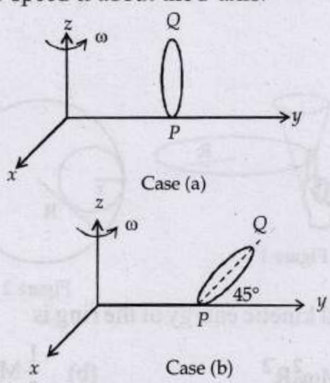
Passage 2

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass.

These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless, stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case **[2012]**



Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



22. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is vertical for both the cases (a) and (b)

(b) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(c) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(d) It is vertical for case (a); and is 45° to the x-z plane and is normal to the plane of the disc for case (b).

23. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is $\sqrt{2}\omega$ for both the cases

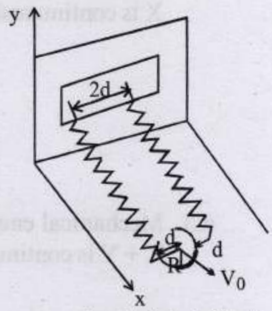
(b) It is ω for case (a); and $\omega/\sqrt{2}$ for case (b)

(c) It is ω for case (a); and $\sqrt{2\omega}$ for case (b)

(d) It is ω for both the cases.

Passage 3

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in horizontal plane.



The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ. **[2008]**

24. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(a) $-kx$ (b) $-2kx$ (c) $-2kx/3$ (d) $-4kx/3$

25. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to –

(a) $\sqrt{\dfrac{k}{M}}$ (b) $\sqrt{\dfrac{2k}{M}}$ (c) $\sqrt{\dfrac{2k}{3M}}$ (d) $\sqrt{\dfrac{4k}{3M}}$

26. The maximum value of V_0 for which the disk will roll without slipping is –

(a) $\mu g\sqrt{\dfrac{M}{k}}$ (b) $\mu g\sqrt{\dfrac{M}{2k}}$ (c) $\mu g\sqrt{\dfrac{3M}{k}}$ (d) $\mu g\sqrt{\dfrac{5M}{2k}}$

Passage 4

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

27. The loss of kinetic energy in the above process is **[2007]**

(a) $\dfrac{I\omega^2}{2}$ (b) $\dfrac{I\omega^2}{3}$ (c) $\dfrac{I\omega^2}{4}$ (d) $\dfrac{I\omega^2}{6}$

28. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is **[2007]**

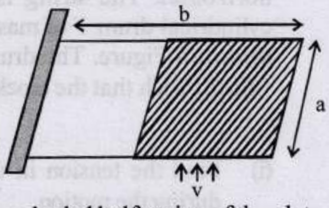
(a) $\dfrac{2I\omega}{3t}$ (b) $\dfrac{9I\omega}{2t}$ (c) $\dfrac{9I\omega}{4t}$ (d) $\dfrac{3I\omega}{2t}$

29. The ratio x_1/x_2 is **[2007]**

(a) 2 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

10 Subjective Problems

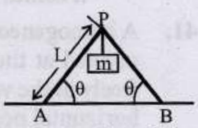
30. A rectangular plate of mass M and dimension $a \times b$ is held in horizontal position by striking n small balls (each of mass m) per unit area per second.



The balls are striking in the shaded half region of the plate. The collision of the balls with the plate is elastic. What is v? **[2006 - 6M]**

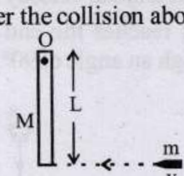
(Given $n = 100$, $M = 3$ kg, $m = 0.01$ kg; $b = 2$ m; $a = 1$ m; $g = 10$ m/s^2).

31. Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P.

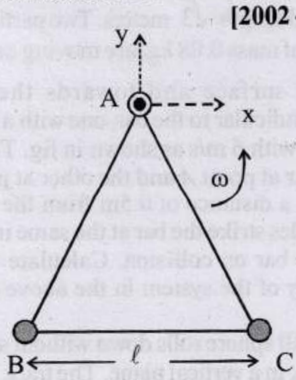


If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B. **[2005 - 4 Marks]**

32. A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it. Find angular velocity of the system immediately after the collision about O. **[2005 - 2 Marks]**



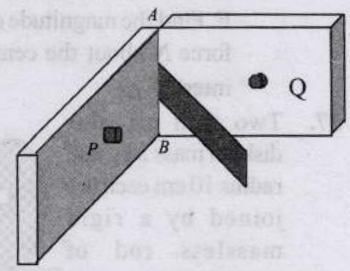
33. Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictioness table (x-y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω. **[2002 - 5 Marks]**



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T.

34. Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its center of mass is 1.2 kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. **[2001-10 Marks]**



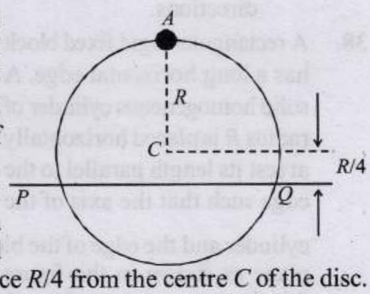
Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s,

(a) Find the location of the center of mass of the laminar sheet from AB.

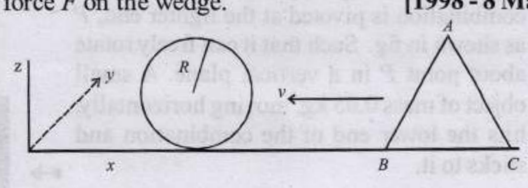
(b) At what angular velocity does the laminar sheet come back after the first impact?

(c) After how many impacts, does the laminar sheet come to rest?

35. A uniform circular disc has radius R and mass m. A particle also of mass m, is fixed at a point A on the edge of the disc as shown in Figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ. Initially, the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle as it reaches its lowest position. **[1998 - 8 Marks]**



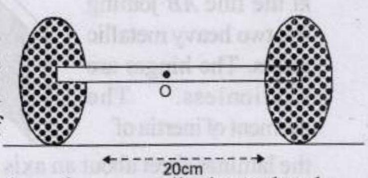
36. A wedge of mass m and triangular cross-section ($AB = BC = CA = 2R$) is moving with a constant velocity $-v\hat{i}$ towards a sphere of radius R fixed on a smooth horizontal table as shown in Figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time. Δt, during which the sphere exerts a constant force F on the wedge. **[1998 - 8 Marks]**



(a) Find the force F and also the normal force N exerted by the table on the wedge during the time Δt.

(b) Let h denote the perpendicular distance between the centre of mass of the wedge and the line of action of **F**. Find the magnitude of the torque due to the normal force **N** about the centre of the wedge, during the interval Δt.

37. Two thin circular disks of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm.



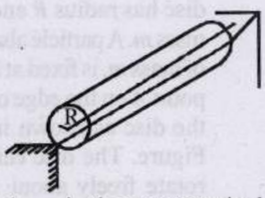
The axis of the rod is along the perpendicular to the planes of the disk through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of the motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x axis as the direction of motion of the truck and z axis as the vertically upwards direction. If the truck has an acceleration of 9 m/s^2.

Calculate: **[1997 - 5 Marks]**

(i) The force of friction on each disk,

(ii) The magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ in the x, y, and z directions.

38. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in the figure below. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine: **[1995 - 10 Marks]**

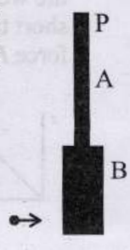


(a) the angle θ_c through which the cylinder rotates before it leaves contact with the edge,

(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and

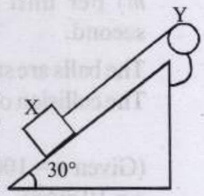
(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

39. Two uniform thin rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in fig. Such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it.



What should be the velocity of the object so that the system could just be raised to the horizontal position. **[1994 - 6 Marks]**

40. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in Figure. The drum is given an initial angular velocity such that the block X starts moving up the plane. **[1994 - 6 Marks]**



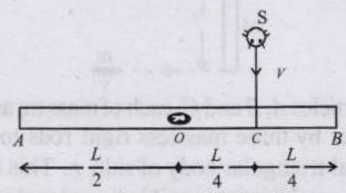
(i) Find the tension in the string during the motion.

(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1} calculate the distance travelled by X from that instant of time until it comes to rest

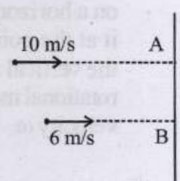
41. A homogeneous rod AB of length $L = 1.8$ m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (Fig). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω. **[1992 - 8 Marks]**

(a) Determine the angular velocity ω in terms of V and L.

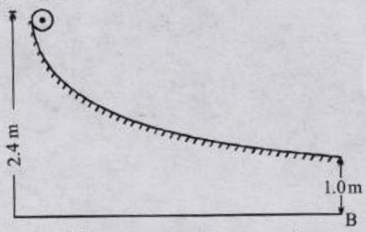
(b) If the insect reaches the end B when the rod has turned through an angle of 90°, determine V.



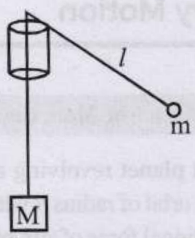
42. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length $\sqrt{3}$ meters. Two particles, each of mass 0.08 kg, are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity of 10 m/s, and other with 6 m/s as shown in fig. The first particle strikes the bar at point A and the other at point B. Points A and B are at a distance of 0.5m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of the kinetic energy of the system in the above collision process. **[1989 - 8 Marks]**



43. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part, The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in fig.) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass? Explain. **[1987 - 7 Marks]**



44. A large mass M and a small mass m hang at two ends of a string that passes over a smooth tube as shown in the figure. The mass m moves around a circular path which lies in a horizontal plane.



The length of string from the mass m to the top of the tube is l and θ is the 'angle' this length makes with the vertical. What should be the frequency of rotation of mass m, so that the mass M remains stationary? **[1978]**

45. A 40 kg mass, hanging at the end of a rope of length l, oscillates in a vertical plane with an angular amplitude θ_0. What is the tension in the rope when it makes an engle θ with the vertical? If the breaking strength of the rope is 80 kg, what is the maximum amplitude with which the mass can oscillate without the rope breaking? **[1978]**

Answer Key

Topic-1 : Centre of Mass, Centre of Gravity & Principle of Moments

1. (b) **2.** (a) **3.** (d) **5.** (False) **6.** (b, c) **7.** (a, c) **8.** (d)

Topic-2 : Angular Displacement, Velocity and Aceleration

1. (a) **2.** (c) **3.** (12) **5.** (a) **6.** (b)

Topic-3 : Torque, Couple and Angular Momentum

1. (a) **2.** (b) **3.** (d) **4.** (c) **5.** (b) **6.** (d) **7.** (b) **8.** (a) **9.** (a) **10.** (b)
11. (b) **12.** (c) **13.** (b) **14.** (c) **15.** (3) **16.** (49) **17.** (4) **18.** (8) **19.** (6) **20.** (25.60)
24. False **25.** (a, b, c) **26.** (a, c, d) **27.** (a, c) **28.** (b, c) **29.** (c) **30.** (a, c, d) **31.** (a, b, d) **32.** (a, c) **33.** (d)
34. (c, d) **35.** (a) **36.** (a, b, c) **37.** (b, d)

Topic-4 : Moment of Inertia and Rotational K.E.

1. (b) **2.** (a) **3.** (a) **4.** (d) **5.** (a) **6.** (18) **7.** (6) **8.** (3) **9.** (9) **11.** (a, b, c)

Topic-5 : Rolling Motion

1. (b) **2.** (d) **3.** (a) **4.** (b) **5.** (c) **6.** (7) **7.** (4) **8.** (0.52) **9.** (2.86)
10. (0.75) **11.** (False) **12.** (a, b, c, d) **13.** (b, d) **14.** (c) **15.** (a, b) **16.** (b, c) **17.** (c, d) **18.** (d)

Topic-6 : Miscellaneous (Mixed Concepts) Problems

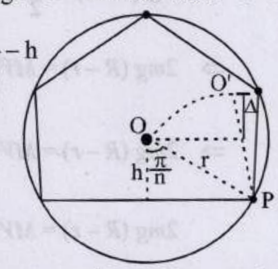
1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (c) **6.** (c) **7.** (c) **8.** (a) **9.** (2) **10.** (0.18)
11. (0.16) **13.** (False) **14.** (a, b, c) **15.** (d) **16.** (c) **17.** (a, c, d) **18.** (a)
19. A → (p, t); B → (q, s, t); C → (p, r, t); D → (q, p) **20.** (c) **21.** (a) **22.** (a) **23.** (d) **24.** (d)
25. (d) **26.** (c) **27.** (b) **28.** (a) **29.** (c)

Hints & Solutions

Topic-1: Centre of Mass, Centre of Gravity & Principle of Moments

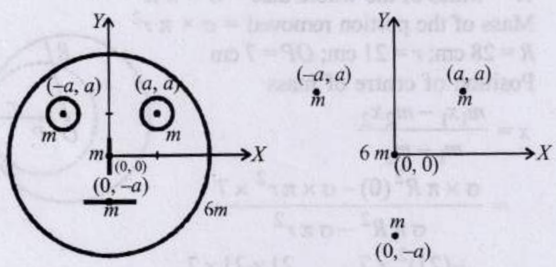
1. **(b)** When the polygon rolls without slipping about the point 'P'. The point O reaches the maximum height (Point O' in the figure) by moving in a circle of radius 'r' about the point 'P'.

So, maximum increase in height of centre of mass 'O' is given by $\Delta = r - h = \dfrac{h}{\cos\left(\dfrac{\pi}{n}\right)} - h$



$\Rightarrow \Delta = h\left(\dfrac{1}{\cos\left(\dfrac{\pi}{n}\right)} - 1\right)$

2. **(a)** The drawing given in the figure is made up of five bodies i.e., three circles and two straight line of uniform mass distribution or we can assume the system to be made up of five point masses where the mass of each is considered at its geometrical centre.



The y-coordinate of the centre of mass is

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4 + m_5y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$\therefore y_{cm} = \frac{6m \times 0 + m \times 0 + m \times a + m \times a + m(-a)}{6m + m + m + m + m}$$

$$= \frac{ma}{10m} = \frac{a}{10}$$

3. **(d)** Mutual force of attraction with which two particles A and B move towards each other is a internal force. There are no external forces acting on the system.

We know $F_{ext} = Ma_{c.m.}$

$a_{c.m.} = 0 \Rightarrow v_{c.m.} = \text{constant.}$

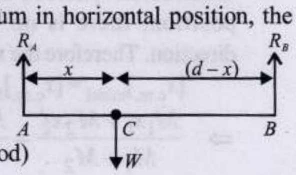
Since, initially $v_{c.m.} = 0$

$\therefore$ Final $V_{cm} = 0$

4. Let R_A = Normal reaction at A

R_B = Normal reaction at B

For the rod to be in equilibrium in horizontal position, the moment of all the forces about A or about B should be zero



$\therefore$ Moment of forces about A,

$xW = R_Bd$ (W = weight of rod)

or $R_B = \dfrac{xW}{d}$ ... (i)

Again, moment of forces about B,

$(d-x)W = R_Ad$ or $R_A = \left(\dfrac{d-x}{d}\right)W$ ... (ii)

$\therefore$ Normal reaction on $A = \left(\dfrac{d-x}{d}\right)W$ and on $B = \dfrac{xW}{d}$.

5. **False.** There is no external force is acting on the two particle system

So, $a_{c.m} = 0$ and hence $V_{c.m} =$ Constant.

6. **(b, c)** Let the block of mass M moves by distance x towards left.

$Mx = m(R - x)$

$\Rightarrow x = \dfrac{mR}{M+m}$ towards left $\therefore x = -\dfrac{mR}{M+m}$

If v is the velocity of mass 'm' as it leaves the block and V is the velocity of block at that instant then according to conservation of linear momentum

mv = MV

By energy conservation

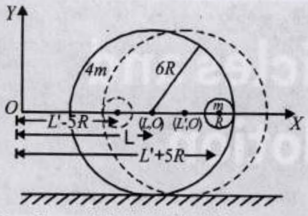
$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Solving we get, $v = \sqrt{\dfrac{2gR}{1 + \dfrac{m}{M}}}$ and $V = \dfrac{m}{M}\sqrt{\dfrac{2gR}{1 + \dfrac{m}{M}}}$

7. **(a, c)** When pedalling stop but the cycle is in motion the wheels move in the direction such that the centre of mass of the wheel move forward. Rolling friction will act in the opposite direction to the relative motion of the centre of mass of the body with respect to ground. Hence the rolling friction will act in backward direction in both the wheels. During pedalling sliding friction will act in the forward direction of rear wheel.

8. **(d)** There is no external torque on a body about its centre of mass, so no Ven is constant. For velocity of centre of mass to remain constant the net force acting on a body must be zero. The linear momentum of an isolated system remains constant.

9. Thin line of sphere represents initial state, dotted line of sphere represents final state.



When small sphere M changes its position to other extreme position, there is no external force in the horizontal direction. Therefore the x-coordinate of c.m. will not change.

$$[x_{c.m.}]_{initial} = [x_{c.m.}]_{final}$$

$$\Rightarrow \frac{M_1x_1 + M_2x_2}{M_1 + M_2} = \frac{M_1x'_1 + M_2x'_2}{M_1 + M_2}$$

$$\Rightarrow \frac{4m \times L + m \times (5R + L)}{4m + m} = \frac{4m \times L' + m \times (L' - 5R)}{4m + m}$$

$$\Rightarrow 5L + 5R = 5L' - 5R$$

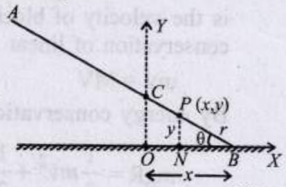
$$\Rightarrow 5L + 10R = 5L' \quad \Rightarrow L + 2R = L'$$

As the individual center of mass of the two spheres has a y co-ordinate zero in its initial state and its final state so, the y-coordinate of c.m. of the two sphere system will remain zero.

Hence, the coordinate of centre of mass of bigger sphere $(L + 2R, 0)$.

10. (i) The rod stands vertical along they yr axis. While rotating, the force acting on the rod are its weight and normal reaction. These forces are vertical forces and cannot create a horizontal motion. Hence the center of mass moves vertically downwards. Thus the path of the center of mass of the rod is a straight line.

(ii) Consider an arbitrary point P on the rod located at (x, y) and at a distance r from the end B. The rod makes an angle θ with the horizontal at this position.



In ΔBNP, $\sin\theta = \frac{y}{r}$ ... (i)

$$\cos\theta = \frac{x + BN}{L/2} = \frac{x + r\cos\theta}{L/2}$$

$$\Rightarrow \cos\theta = \frac{x}{\frac{L}{2} - r} \quad \text{... (ii)}$$

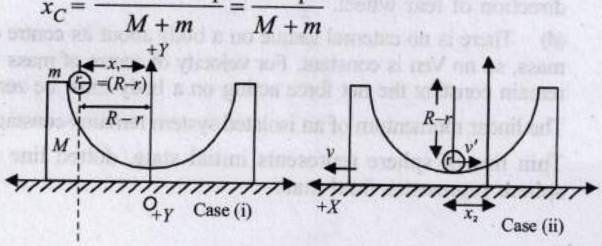
$$\because \sin^2\theta + \cos^2\theta = 1$$

$$\therefore \frac{y^2}{r^2} + \frac{x^2}{\left(\frac{L}{2} - r\right)^2} = 1$$

Hence shape of the path of this point is an ellipse.

11. Case (i) centre of mass of the system of two bodies in x-coordinate

$$x_C = \frac{M \times 0 + mx_1}{M + m} = \frac{mx_1}{M + m}$$



Case (ii) centre of mass of the system in x-coordinate

$$x'_C = \frac{M \times x_2 + m \times x_2}{M + m} = x_2$$

$$x_C = x'_C \quad (\because \text{ Fexternal} = 0 \text{ in } x\text{-direction})$$

$$\therefore x_2 = \frac{mx_1}{M + m} = \frac{m(R - r)}{M + m}$$

i.e., the block more $x_2 = \frac{m(R - r)}{M + m}$ when

The cylinder reaches the bottom point β of the track

Applying conservation of linear momentum, $P_i = P_f$

$$0 = MV - mv$$

$$\therefore v = \frac{MV}{m}$$

Applying conservation of energy,

$$\Rightarrow mg(R - r) = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$$

$$\Rightarrow 2mg(R - r) = MV^2 + m\frac{M^2V^2}{m^2} \quad [\because v = \frac{MV}{m}]$$

$$\Rightarrow 2mg(R - r) = MV^2 + \frac{M^2V^2}{m}$$

$$2mg(R - r) = MV^2\left[1 + \frac{M}{m}\right] = MV^2\left[\frac{m + M}{m}\right]$$

$$\Rightarrow \frac{2m^2 g(R - r)}{M(m + M)} = V^2 \Rightarrow V = m\sqrt{\frac{2g(R - r)}{M(m + M)}}$$

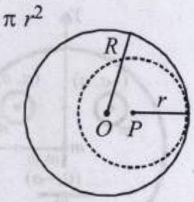
12. Let σ be the mass per unit area of uniform plate.

$\therefore$ Mass of the whole disc $= \sigma \times \pi R^2$

Mass of the portion removed $= \sigma \times \pi r^2$

$R = 28$ cm; $r = 21$ cm; $OP = 7$ cm

Position of centre of mass



$$x = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

$$= \frac{\sigma \times \pi R^2(0) - \sigma \times \pi r^2 \times 7}{\sigma\pi R^2 - \sigma\pi r^2}$$

$$= \frac{-(21)^2 \times 7}{(28)^2 - (21)^2} = \frac{21 \times 21 \times 7}{7 \times 49} = -9\text{ cm}$$

i.e., centre of mass lies at a distance of 9 cm from the origin towards left.

Topic-2: Angular Displacement, Velocity and Acceleration

1. (a) At $t = 0$, $t = \frac{T}{2}$ and $t = T$ the relative velocity will be zero.

At $t = \frac{T}{4}$ and $t = \frac{3T}{4}$, the relative velocity will be maximum in magnitude

Hence graph (a) correctly depicts v_r versus t graph.

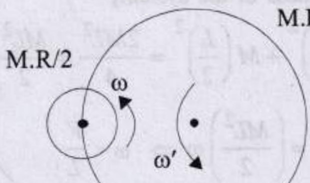
2. (c) At $t = \frac{1}{8} \times \frac{2\pi}{\omega} = \frac{\pi}{4\omega}$

$$x - \text{coordinate of P} = \omega R\left(\frac{\pi}{4\omega}\right) = \frac{\pi R}{4} > R\cos 45°$$

Therefore, both particles P and Q land in unshaded region.

3. **(12)** Using conservation of angular momentum of the system (bigger + smaller disc) about the symmetric axis of large disc.

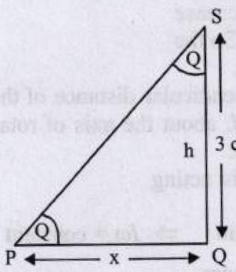
$$\frac{1}{2}M\left(\frac{R}{2}\right)^2\omega - M(\omega' R).R - \frac{MR^2}{2}.\omega' = 0$$



$$\Rightarrow \frac{\omega}{8} = \frac{3\omega'}{2} \Rightarrow \omega' = \frac{\omega}{12}$$

Hence, the value of n = 12

4.



The velocity (v) of spot $= dx/dt$

and the angular speed (ω) of spot light $= \frac{d\phi}{dt}$

From Δ SOP,

$$\tan\phi = \frac{x}{h} \Rightarrow x = h\tan\phi$$

$$\therefore \frac{dx}{dt} = h\sec^2\phi\frac{d\phi}{dt} \quad \text{or, } v = (h\sec^2\phi)\,\omega$$

$$\therefore v = 3\sec^2 45° \times 0.1 \qquad [\because \theta + \phi = 90°]$$

$$\therefore v = 3 \times 2 \times 0.1 = 0.6 \text{ m/s}$$

5. **(a)** Force on the block along slot $= m r\omega^2 = ma = m v\frac{dv}{dr}$

$$\therefore \int_0^v V dv = \int_{R/2}^{r}\omega^2 r dr \Rightarrow V = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\therefore \int_{R/4}^{r}\frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega\, dt$$

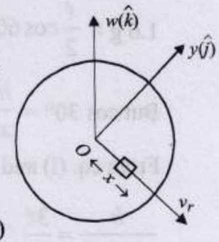
On solving we get $r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$

$$\text{or } r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t} \quad \therefore r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

6. **(b)** $\vec{F}_{rot} = \vec{F}_{in} + 2m\left(V_{rot}\,\hat{i}\right)\times\omega\hat{k} + m\left(\omega\hat{k}\times r\hat{i}\right)\times\omega\hat{k}$

$$\therefore m r\omega^2\,\hat{i} = \vec{F}_{in} + 2m\,V_{rot}\,\omega\left(-\hat{j}\right) + m\omega^2 r\,\hat{i}$$

$$\vec{F}_{in} = mrV_{rot}\,\omega\,\hat{j} \qquad \text{.... (i)}$$



But $r = \frac{R}{4}\left[e^{\omega t} + e^{-\omega t}\right]$

$$\therefore \frac{dr}{dt} = V_r = \frac{R}{4}\left[\omega e^{\omega t} - \omega e^{-\omega t}\right] \text{....(ii)}$$

From (i) and (ii)

$$\vec{F}_{in} = 2m\frac{R\omega}{4}\left(e^{\omega t} - e^{-\omega t}\right)\omega\,\hat{j}$$

$$\therefore \vec{F}_{in} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

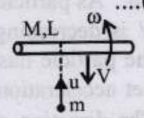
$$\therefore \vec{F}_{reaction} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\,\hat{K}$$

Topic-3: Torque, Couple and Angular Momentum

1. **(a)** About the hinge applying angular momentum conservation

$$mv\frac{L}{2} + 0 = -mv\frac{L}{2} + \frac{ML^2}{3}\omega \qquad \text{....(i)}$$

After elastic collision,



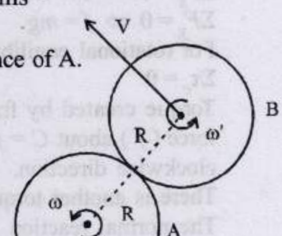
$$e = 1 = \frac{\omega\frac{L}{2} + V}{u} \Rightarrow u = \omega\frac{L}{2} + V \qquad \text{....(ii)}$$

Putting, m = 0.1 Kg, M = 1 Kg, L = 0.20 m and solving eq (i) & (ii) we get

$\omega \approx 6.98 \text{ rad s}^{-1}$ and $v = 4.30 \text{ ms}^{-1}$

2. **(b)** We have V = (2R) ω

As B is rolling on circumference of A.



So, V = ω'R

$\Rightarrow$ 2Rω = ω'R

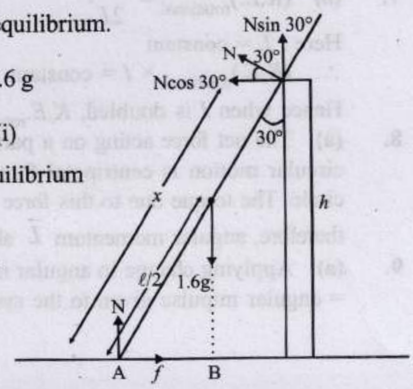
$\Rightarrow$ ω' = 2ω

$$\text{So, } \left(\vec{L}\right)_{\text{w.r.t COM of A}} = \left(I\omega' + m(2R)V\right)\hat{k}$$

$$= \left(\frac{1}{2}MR^2(2\omega) + m(2R)(2R\omega)\right)\hat{k} = 5MR^2\omega\,\hat{k}$$

So, n = 5

3. **(d)** By vertical equilibrium.



$$N + N\sin 30° = 1.6\,g$$

$$\Rightarrow N = \frac{3.2g}{3} \text{ ... (i)}$$

By horizontal equilibrium

$$f = N\cos 30°$$

$$= \frac{\sqrt{3}}{2}N = \frac{16\sqrt{3}}{3}$$

Torque about A

$1.6g \times AB = N \times x$

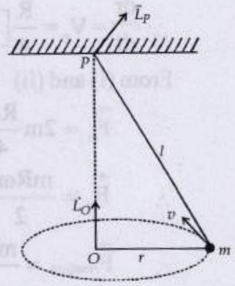
$1.6\,g \times \frac{\ell}{2}\cos 60° = \frac{3.2g}{3} \times x \quad \therefore \frac{3\ell}{8} = x \quad \ldots (i)$

But $\cos 30° = \frac{h}{x} \quad \therefore x = \frac{h}{\cos 30°} \ldots (ii)$

From eq. (i) and (ii)

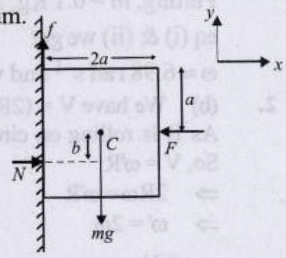
$\frac{h}{\cos 30°} = \frac{3\ell}{8} \quad \therefore \frac{h}{\ell} = \frac{3\sqrt{3}}{16}$

4. **(c)** For all locations of m the angular momentum of the mass m about O i.e., L_o is $mr^2\omega$ and is directed toward $+z$ direction. The angular momentum of mass m about P i.e., L_p is mvl and is directed for the given location of m as shown in the figure. For different location of m, the direction of $\vec{L}_P$ remains changing.



5. **(b)** As particle is rotating in circular path and linear speed V is decreasing, L is not conserved in magnitude. Since the particle has two accelerations a_c and a_t therefore the net acceleration is not towards the centre.
The direction of $\vec{L}$ remains same even when the speed decreases.

6. **(d)** The block is in equilibrium.
For translational equilibrium
$\Sigma F_x = 0 \Rightarrow F = N$
$\Sigma F_y = 0 \Rightarrow f = mg$
For rotational equilibrium
$\Sigma \tau_c = 0$
Torque created by frictional force (f) about $C = f \times a$ in clockwise direction.



There is another torque which should counter this torque. The normal reaction N on the block will create a torque $N \times b$ in the anticlockwise direction.
Such that $f \times a = N \times b$
Hence N will produce torque.

7. **(b)** $(K.E.)_{rotational} = \frac{L^2}{2I}$.
Here, L = constant
$\therefore \quad (K.E.)_{rotational} \times I$ = constant.
Hence when I is doubled, $K.E._{rotational}$ becomes half.

8. **(a)** The net force acting on a particle undergoing uniform circular motion is centripetal force cowards centre of the circle. The torque due to this force about the centre is zero, therefore, angular momentum $\vec{L}$ about centre is conserved.

9. **(a)** Applying change in angular momentum of the system = angular impulse given to the system about the centre of mass (Angular momentum)$_f$ − (Angular momentum)$_i$

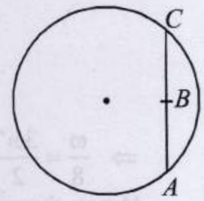
$= Mv \times \frac{L}{2} = I_C\omega \quad \ldots (i)$

Here ω is the angular velocity of the rod.
Moment of Inertia of the system about its axis of rotation [centre of mass of the system]

$I_C = M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{2ML^2}{4} = \frac{ML^2}{2}$

$\therefore \quad \frac{MVL}{2} = \left(\frac{ML^2}{2}\right)\omega \Rightarrow \omega = \frac{V}{L}$

10. **(b)** There is no external torque, angular momentum remains conserved. As moment of inertia initially when the tortoise moves from A to B decreases and then increases when tortoise moves from B to C, so ω will increase initially and then decreases. So the variation of a ω is nonlinear.



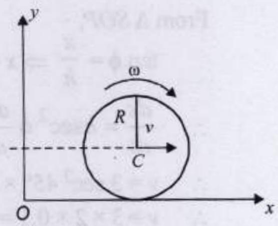
11. **(b)** Due to increase in perpendicular distance of the bead with the axis of rotation $M.I.$ about the axis of rotation is not constant. So I increases.
There is no external torque is acting

$\therefore \tau_{ext} = \frac{dL}{dt} \Rightarrow L = \text{constant} \quad \Rightarrow I\omega = \text{constant}$

But, I increases so ω decreases.

12. **(c)** When a disc rolls, it has two types of motion i.e., translational and rotational. Therefore there are two types of angular momentum and the total angular momentum is the vector sum of these two.

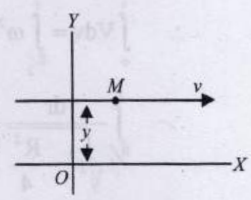
$\vec{L} = \vec{L}_T + \vec{L}_R$
L_T = angular momentum due to translational motion.
L_R = angular momentum due to rotational motion about C.M.
$L = MV \times R + I_{cm}\omega$



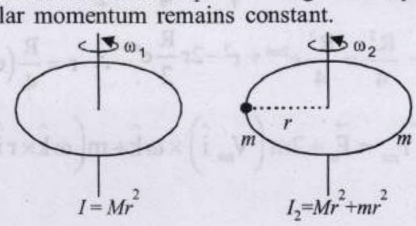
I_{cm} = M.I. about centre of mass C and $V = R\omega$

$= M(R\,\omega)\,R + \frac{1}{2}MR^2\omega = \frac{3}{2}MR^2\omega$

13. **(b)** Angular momentum
$\vec{L} = r\perp \times \vec{p}$
$L = Mv \times y$
According to question, M, v and y constant hence its angular momentum w.r.t. origin remains constant.



14. **(c)** Since the two objects are placed gently to the ring, therefore no external torque is acting on the system. Hence angular momentum remains constant.



$\therefore \quad I_1\omega_1 = I_2\omega_2$

$\Rightarrow \quad Mr^2 \times \omega_1 = (Mr^2 + 2mr^2)\,\omega_2 \quad (\because \omega_1 = \omega)$

$\therefore \quad \omega_2 = \dfrac{M\omega}{M+2m}$

15. **(3)** Here, $\vec{F} = -K\vec{r}$. So force passes through origin.
So, $\tau_{origin} = 0 \Rightarrow$ angular momentum about origin will be conserved

So, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & \frac{2}{\pi} \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0.5 \\ v_x & v_y & \frac{2}{\pi} \end{vmatrix}$

$\Rightarrow \hat{k}\left[\dfrac{1}{\sqrt{2}} \times \sqrt{2} - (-\sqrt{2}) \times \sqrt{2}\right] = \hat{k}(xv_y - yv_x)$

$\Rightarrow xv_y - yv_x = 3$

16. **(49)** Angular momentum of the system, $L_s = L_{rod} + L_{disc}$

$= I_{rod}\omega + I_{disc}\omega + \vec{r} \times \vec{P}$

$= \dfrac{Ma^2}{12}\omega + \dfrac{Ma^2\omega}{32} + \dfrac{3a}{16} \times \dfrac{3a}{4} \times M \times \omega$

$= \dfrac{Ma^2}{12} \times 4\Omega + \dfrac{Ma^2}{32} \times 4\Omega + \dfrac{3a}{16} \times \dfrac{3a}{4} \times M \times 4\Omega$

$\therefore \quad L_s = \dfrac{Ma^2}{3}\Omega + M\left(\dfrac{3a}{4}\right)^2 \Omega + \dfrac{M\left(\frac{a}{4}\right)^2 4\Omega}{2}$

or $\quad L_s = \dfrac{49}{48} Ma^2\Omega \qquad \therefore \quad n = 49$

17. **(4)** From conservation of angular momentum
$2\,(mvr) = I\omega$

$2 \times 0.05 \times 9 \times 0.25 = \dfrac{1}{2} \times 0.45 \times (0.5)^2 \times \omega$

$\therefore \quad \omega = 4 \text{ rad s}^{-1}$

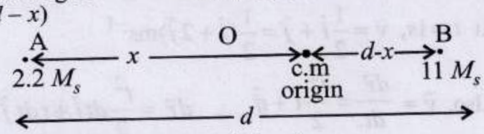
18. **(8)** From conservation of angular momentum
$I_1\omega_1 = I_2\omega_2$

$\therefore \omega_2 = \dfrac{I_1\omega_1}{I_2} = \dfrac{\frac{1}{2}MR^2 \times \omega_1}{\left\{\frac{1}{2}MR^2 + 2[2mr^2]\right\}}$

Given: $M = 50$ kg, $R = 0.4$ m, $\omega_1 = 10$ rod/s
$m = 6.25$ kg and $r = 0.2$ m

$= \dfrac{\frac{1}{2} \times 50 \times 0.4 \times 0.4 \times 10}{\frac{1}{2} \times 50 \times 0.4 \times 0.4 + 2[2 \times 6.25 \times 0.2 \times 0.2]} = \dfrac{40}{4+1} = 8 \text{ rad/s}$

19. **(6)** Let the center of mass of the binary star system be at the origin. '*O*'. Distance between $AO = x$ and $OB = (d-x)$



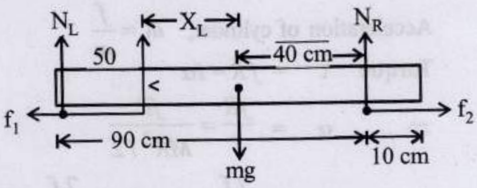
$0 = \dfrac{2.2M_s(-x) + 11M_s(d-x)}{2.2M_s + 11M_s}$

$\Rightarrow 0 = 2.2\,M_s(-x) + 11M_s(d-x) \;\Rightarrow x = \dfrac{5d}{6}$

For a binary star system, angular speed ω about the centre of mass. 'O' is same for both the stars.

$\therefore \dfrac{L_{Total}}{L_B} = \dfrac{2.2M_s\left(\frac{5d}{6}\right)^2 \omega + 11M_s\left(\frac{d}{6}\right)^2 \times \omega}{11M_s\left(\frac{d}{6}\right)^2 \times \omega} = 6$

20. **(25.60)**



Initially

$N_L + N_R = Mg \qquad N_L = \dfrac{40}{90}Mg = \dfrac{4Mg}{9}$

Torque about centre $\tau_{center} = 0$

$\therefore N_1(50) = N_2(40) \quad N_R = \dfrac{50}{90}Mg = \dfrac{5Mg}{9}$

$5N_L = 4N_R$

$f_{1K} = \mu_K N_L \qquad f_{1L} = \mu_S N_L$

$f_{1K} = 0.32N_L \qquad f_{1L} = 0.4N_L$

$f_{2K} = 0.32N_R \qquad f_{2L} = 0.4N_R$

If X_L = distance of left finger from centre when right finger starts moving

$(\tau_n = 0)_{\text{about centre}} \Rightarrow N_{LX_L} = N_R(40)$

$f_{K_1} = f_{L_2} \qquad \Rightarrow 0.32N_L = 0.40N_R$

$4N_L = 5N_R$

$N_{LX_L} = \dfrac{4N_L}{5}(40) \Rightarrow X_L = 32$

Now x_R = distance when right finger stops and left finger starts moving

$N_{LX_L} = N_R(x_R) \qquad$ [Torque about centre, $T_{centre} = 0$]

$f_{L_1} = f_{K_2} \qquad \Rightarrow 0.4N_L = 0.32N_R$

$4N_L = 4N_R$

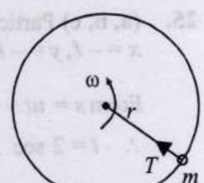
$\dfrac{4N_2}{5}(32) = N_R X_R$

$\therefore x_R = \dfrac{128}{5} = 25.6$ cm

21. Let at any instant of time *t*, the radius of the horizontal surface = *r*.

$T = \dfrac{mv^2}{r} = mr\omega^2 \qquad$ ... (i)

Where *m* is the mass of stone and ω is the angular velocity at that instant of time *t*.



Also, $L = I\omega \Rightarrow w = \dfrac{L}{1}$

Putting this value of *w* is eq.(i)

$T = \dfrac{mrL^2}{I^2} = \dfrac{mL^2}{(mr^2)^2} \times r \Rightarrow T = \left(\dfrac{L^2}{m}\right) r^{-3}$

or, $T = Ar^{-3} \;\therefore\; n = -3 \qquad \left(\dfrac{L^2}{m} = A \text{ constant}\right)$

22. Considering the motion of the platform
$x = A\cos\omega t \qquad$ (given)

$\Rightarrow \frac{dx}{dt} = -A\omega \sin \omega t \Rightarrow \frac{d^2x}{dt^2} = -A\omega^2 \cos \omega t$

$\therefore$ Magnitude of the maximum acceleration of the platform

$\therefore$ | Max acceleration | $= A\omega^2$

When platform moves a torque acts on the cylinder and the cylinder rotates about its axis.

Acceleration of cylinder, $a_1 = \frac{f}{m}$

Torque $\tau = fR = I\alpha$

$\Rightarrow \alpha = \frac{fR}{I} = \frac{fR}{MR^2/2}$

or, $\alpha = \frac{2f}{MR}$ or $R\alpha = \frac{2f}{M} (= a_2)$

$\therefore$ Total or max linear acceleration,

$$a_{max} = a_1 + a_2 = \frac{f}{M} + \frac{2f}{M} = \frac{3f}{M}$$

or, $A\omega^2 = \frac{3f}{M} \Rightarrow f = \frac{MA\omega^2}{3}$

Thus, maximum torque,

$$\tau_{max} = f \times R = \frac{MA\omega^2 R}{3} = \frac{1}{3} MAR\omega^2$$

23. There is no external force and hence no torque is applied, the angular momentum remains constant *i.e.*; $I_1 = I_2$

$\therefore I_1\omega_1 = I_2\omega_2$

$$\therefore \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\frac{ML^2}{12} \times \omega_0}{\frac{ML^2}{12} + 2m \times \left(\frac{L}{2}\right)^2} = \frac{M\omega_0}{M + 6m}$$

24. False

$I_1\omega_1 = I_2\omega_2$ $(\because \vec{L} = \text{constant})$

$I_1 = \frac{1}{2}MR^2$

$I_2 = \frac{1}{2}MR^2 + \frac{1}{2}\frac{M}{4}R^2 = \left(\frac{4+1}{8}\right)MR^2 = \frac{5}{8}MR^2$

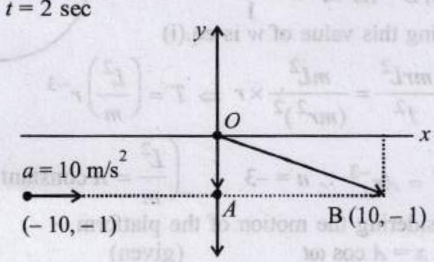
$\omega_1 = \omega$; $\omega_2 = ?$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = +\frac{\frac{1}{2}MR^2 \times \omega}{\frac{5}{8}MR^2} = \frac{8}{2 \times 5}\omega = \frac{4}{5}\omega$$

25. (a, b, c) Particle is initially at rest in the *xy*-plane at a point $x = -\ell, y = -h$ where $\ell = 10$ m and $h = 1$ m.

From $s = ut + \frac{1}{2}at^2 \Rightarrow 20 = \frac{1}{2} \times 10 \times t^2$

$\therefore t = 2$ sec



Torque, $\vec{\tau} = \vec{r} \times \vec{F}$ and here $\vec{r}_B = 10\hat{i} - \hat{j}$

$\vec{F} = m\vec{a} = 0.2 \times 10\hat{i} = 2\hat{i}$ $\therefore \vec{\tau} = (10\hat{i} - \hat{j}) \times (2\hat{i}) = 2\hat{k}$

Angular momentum, $\vec{L} = \vec{r}_B \times \vec{P} = \vec{r}_B \times m\vec{v}$

From $\vec{v} = u + \vec{a}t = 10\hat{i} \times 2 = 20\hat{i}$

$\vec{L} = (0.2)\left[(10\hat{i} - \hat{j}) \times 20\hat{i}\right] = 4\hat{k}$

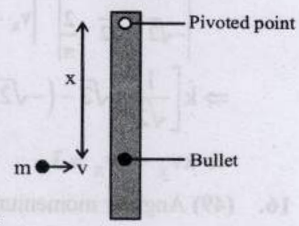
At point A (0, – 1)

Torque $\vec{\tau} = \vec{r}_A \times \vec{F} = (-\hat{j}) \times 2\hat{i} = 2\hat{k}$ $\left[\because \vec{r}_A = -\hat{j}\right]$

26. (a, c, d) From angular momentum conservation about the pivoted point.

$$mvx = \left(\frac{mL^2}{3} + mx^2\right)\omega$$

[As the combined system rotates with angular speed ω about the pivot]



$$\therefore \omega = \frac{mvx}{\frac{mL^2}{3} + mx^2} = \frac{3vx}{L^2 + 3x^2}$$

$$\omega = \frac{3vx}{L^2 + 3x^2}$$

Hence option (a) is correct.

For maximum angular velocity, $\frac{d\omega}{dx} = 0$

$$\frac{d}{dx}\left(\frac{L^2}{x} + 3x\right) = 0 \Rightarrow \frac{L^2}{x^2} + 3 = 0 \Rightarrow x = \frac{L}{\sqrt{3}}$$

$\therefore X_m = \frac{L}{\sqrt{3}}$

So option (c) is correct.

$$\omega_m = \frac{3vx}{L^2 + 3x^2} = \frac{3v\frac{L}{\sqrt{3}}}{L^2 + 3\left(\frac{L}{\sqrt{3}}\right)^2} = \frac{\sqrt{3}}{2L}V$$

Hence option (d) is correct.

27. (a, c) Given $\vec{F} = \alpha t\hat{i} + \beta\hat{j}$ or $\vec{F} = t\hat{i} + \hat{j}$

$(\because \alpha = 1NS^{-1}$ and $\beta = 1N)$

$\therefore \frac{m\,d\vec{v}}{dt} = t\,\hat{i} + \hat{j}$

$\therefore d\vec{v} = t\,dt\,\hat{i} + dt\,\hat{j}$ $[\because m = 1]$

$\therefore \int_0^v d\vec{v} = \int_0^t t\,dt\,\hat{i} + \int_0^t dt\,\hat{j} \Rightarrow \vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

At $t = 1$s, $\vec{v} = \frac{1}{2}\hat{i} + \hat{j} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

Also, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$ $\therefore d\vec{r} = \frac{t^2}{2}dt\,\hat{i} + t\,dt\,\hat{j}$

or, $\int_0^r d\vec{r} = \int_0^t \frac{t^2}{2}dt\,\hat{i} + \int_0^t t\,dt\,\hat{j}$ $\therefore \vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

At $t = 1$, $\vec{r} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$ $\therefore |\vec{r}| = \sqrt{\frac{1}{36} + \frac{1}{4}} = \sqrt{\frac{10}{36}}$

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j}) \quad (\text{at } t = 1\text{s})$$

or, $\vec{\tau} = -\frac{1}{3}\hat{k} \quad \therefore \quad |\vec{\tau}| = \frac{1}{3}$ Nm

28. **(b, c)** Given : potential energy, $V(r) = Kr^2/2$

Applying, $|F| = \frac{dv}{dr} = \frac{d}{dr}\left[\frac{kr^2}{2}\right] = kr$

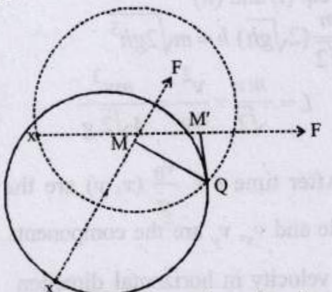
For $r = R$, $F = kR$

Also $F = \frac{mv^2}{R}$ ($\because$ Particle is moving in circular orbit)

or, $\frac{mv^2}{R} = kR \Rightarrow v = \sqrt{\frac{k}{m}} \times R$

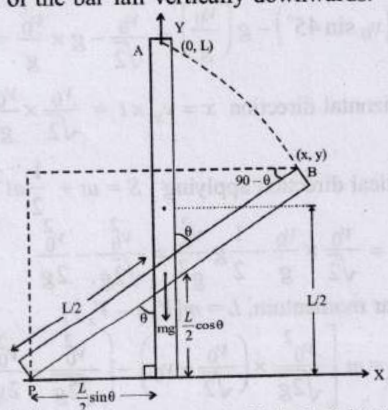
And angular momentum, $L = mvR = m\left(\sqrt{\frac{k}{m}}R\right)R = \sqrt{km}R^2$

29. **(c)** If the force (F) is applied at P tangential than the τ remains constant and $\tau = F \times 2R$.
If force is applied normal to X, then as the wheels climbs, then the perpendicular distance of force from Q will go on changing initially the perpendicular is QM, later it becomes QM'.



If the force (F) is applied normal to the circumference at point P then τ is zero.
If the force (F) is applied tangentially at point S then $\tau = F \times R$ and the wheel climbs.

30. **(a, c, d)** Force acting on com $F_x = 0$, $\therefore a_x = 0$. Therefore the force acting in vertical direction will move the mid point or com of the bar fall vertically downwards.



When the bar makes an angle θ the height of its com $= \frac{L}{2}\cos\theta$

$\therefore$ Displacement of its mid point from the initial position

$$\frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$$

Instantaneous torque about the point of contact P

$$\tau = mg \times \frac{L}{2}\sin\theta$$

Now $x = \frac{L}{2}\sin\theta$, $\quad y = L\sin(90° - \theta) = L\cos\theta$

$$\therefore \left(\frac{2x}{L}\right)^2 + \left(\frac{y}{L}\right)^2 = 1 \text{ or } \frac{4x^2}{L^2} + \frac{y^2}{L^2} = 1$$

Thus path of A is an ellipse not parabola.

31. **(a, b, d)** Given: $\vec{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$$\vec{r} = \frac{10}{3}t^3\hat{i} + 5t^2\hat{j}m$$

$$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = 10t^2\hat{i} + 10t\hat{j}\ ms^{-1}$$

and $\vec{a} = \frac{d\vec{v}}{dt} = 20t\hat{i} + 10\hat{j}ms^{-2}$

At t = 1s $\vec{r}_{t=1} = \frac{10}{3}\hat{i} + 5\hat{j}m$;

$\vec{v}_{t=1} = 10\hat{i} + 10\hat{j}\ ms^{-1}$ and $\vec{a}_{t=1} = 20\hat{i} + 10\hat{j}\ ms^{-2}$

$\vec{p}_{t=1} = \hat{i} + \hat{j}\ kgms^{-1}$

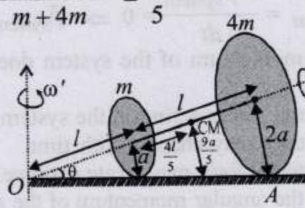
$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10}{3} & 5 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{k}\left[\frac{10}{3} - 5\right] = -\frac{5}{3}\hat{k}\text{ kgms}^{-1}$$

$$\vec{F} = m\vec{a} = (2\hat{i} + \hat{j})N$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10}{3} & 5 & 0 \\ 2 & 1 & 0 \end{vmatrix} = \hat{k}\left[\frac{10}{3} - 10\right] = \frac{-20}{3}\hat{k}\text{ Nm}$$

32. **(a, c)** Position of CM on the axis of rod.

$$x_{CM} = \frac{m(0) + 4m(l)}{m + 4m} = \frac{4l}{5}$$



$$\cos\theta = \frac{1}{\sqrt{l^2 + a^2}} = \frac{\sqrt{24}a}{\sqrt{a^2 + 24a^2}} = \frac{\sqrt{24}}{5}$$

$[\because l = \sqrt{24}a \text{ given}]$

$$OA = \sqrt{(2l)^2 + (2a)^2} = \sqrt{96a^2 + 4a^2} = 10a$$

Let complete system rotates about z-axis with a constant angular velocity ω'

$$\therefore \quad \frac{\omega'}{\omega} = \frac{2\pi(2a)}{2\pi(10a)} \Rightarrow \omega' = \frac{\omega}{5}$$

Magnitude of angular momentum of the system about its center of mass

$$L_{CM} = I_{CM}\omega = \left[\frac{ma^2}{2} + \frac{4m(2a)^2}{2}\right]\omega = \frac{17}{2}ma^2\omega$$

Magnitude of angular momentum of CM of system about point O.

$$L' = 5m \times \frac{9l\omega}{5} \times \frac{9a}{5} = \frac{81m\omega la}{5} = \frac{81\sqrt{24}m\omega a^2}{5}$$

Magnitude of z-component of angular momentum of system about point 0.

$L_z = L'\cos\theta - L_{CM}\sin\theta$

$$= \frac{81\sqrt{24}m\omega a^2}{5} \times \frac{\sqrt{24}}{5} - \frac{17}{5}ma^2\omega \times \frac{1}{5}$$

$$= ma^2\omega\left(\frac{1944}{25} - \frac{17}{10}\right)$$

33. **(d)** Applying conservation of angular mumentum about the axis

$$MR^2 \times \omega = MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8}r^2 \times \frac{8\omega}{9}$$

Solving we get

$\Rightarrow \quad r = \frac{4R}{5}$

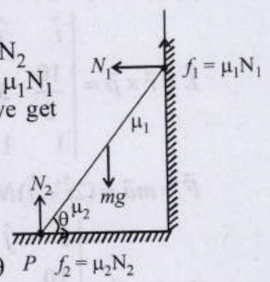
34. **(c, d)** When $\mu_1 \neq 0$ and $\mu_2 \neq 0$

Horizontal equilibrium, $N_1 = \mu_2 N_2$

Vertical equilibrium, $mg = N_2 + \mu_1 N_1$

Solving the above equations we get

$$N_2 = \frac{mg}{1+\mu_1\mu_2}$$



When $\mu_1 = 0$; Torque about P

$$mg \times \frac{l}{2}\cos\theta = N_1 \times l\sin\theta$$

$$\Rightarrow \quad N_1 \tan\theta = \frac{mg}{2}$$

35. **(a)** Given $\sum \vec{F}_{ext} = 0$

$$\sum \vec{F}_{ext} = \frac{d\,\vec{p}_{system}}{dt} = 0 \Rightarrow \vec{p}_{system} = \text{constant}$$

i.e., Linear momentum of the system does not change in time.

Due to internal forces acting in the system, the kinetic and potential energy may change with time.

Also zero external force may create a torque. Thus the torque will change the angular momentum of the system in time.

36. **(a, b, c)** Given $\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L} \quad \left(\because \vec{\tau} = \frac{d\vec{L}}{dt}\right)$

From cross-product rule, $\frac{d\vec{L}}{dt}$ is perpendicular to both $\vec{A}$ and $\vec{L}$.

From dot product rule, $\vec{L}.\vec{L} = L^2$

Differentiating with respect to time

$$\vec{L}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt} \Rightarrow 2\vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

Since, $\quad \vec{L} \perp \frac{d\vec{L}}{dt}$

$$\therefore \vec{L}\cdot\frac{d\vec{L}}{dt} = 0 \Rightarrow \frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$$

Thus, the magnitude of L always remains constant.

As $\vec{A}$ is a constant vector and it is always perpendicular to $\vec{\tau}$,

Also, $\vec{L}$ is perpendicular to $\vec{A}$

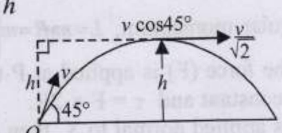
$\because \vec{L} \perp \vec{A} \quad \therefore \vec{L}.\vec{A} = 0$

Hence, it can be concluded that component of $\vec{L}$ along $\vec{A}$ is zero i.e., constant.

37. **(b, d)** Angular momentum $L = r_\perp \times P$

Angular momentum about point O

$$L = \frac{mv}{\sqrt{2}} \times h \quad \text{...(i)}$$



$$\text{Also, } h = \frac{v^2\sin^2\theta}{2g} = \frac{v^2}{4g} \quad [\because \theta = 45°] \quad \text{...(ii)}$$

From eq. (i) and (ii)

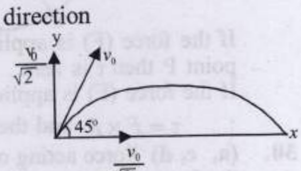
$$L = \frac{m}{\sqrt{2}}(2\sqrt{gh})\,h = m\sqrt{2gh^3}$$

$$\text{Also} \quad L = \frac{mv}{\sqrt{2}} \times \frac{v^2}{4g} = \frac{mv^3}{4\sqrt{2}\,g}$$

38. Let, After time $t = \frac{v_0}{g}$ (x, y) are the coordinates of the particle and v_x, v_y are the components of velocities at that time.

Here, velocity in horizontal direction

$$v_x = v_0\cos 45° = \frac{v_0}{\sqrt{2}}$$



Applying $v = u + at$ in the vertical direction

$$v_y = (v_0\sin 45°) - g\left(\frac{v_0}{g}\right) = \frac{v_0}{\sqrt{2}} - g \times \frac{v_0}{g} = \frac{v_0}{\sqrt{2}} - v_0$$

In horizontal direction $x = v_x \times t = \frac{v_0}{\sqrt{2}} \times \frac{v_0}{g} = \frac{v_0^2}{\sqrt{2}g}$

In vertical direction applying $S = ut + \frac{1}{2}at^2$

$$y = \frac{v_0}{\sqrt{2}} \times \frac{v_0}{g} - \frac{1}{2}g\frac{v_0^2}{g^2} = \frac{v_0^2}{\sqrt{2}g} - \frac{v_0^2}{2g}$$

Angular momentum, $L = m[V_y x - V_x y]$

$$\text{or,} \quad L = m\left[\frac{v_0^2}{\sqrt{2}g} \times \left(\frac{v_0}{\sqrt{2}} - v_0\right) - \left(\frac{v_0^2}{\sqrt{2}g} - \frac{v_0^2}{2g}\right)\frac{v_0}{\sqrt{2}}\right]$$

$$L = m\left[\frac{v_0^3}{2g} - \frac{v_0^3}{\sqrt{2}g} - \frac{v_0^3}{2g} + \frac{v_0^3}{2\sqrt{2}\,g}\right]$$

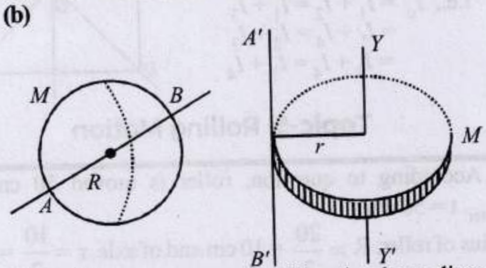
$$L = \frac{mv_0^3}{g}\left[\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right] = \frac{-mv_0^3}{2\sqrt{2}\,g}$$

Again, $\vec{L} = \vec{r} \times \vec{p}$

Hence, the direction of L is perpendicular to the plane of motion and is directed away from the reader.

Topic-4: Moment of Inertia and Rotational K.E.

1. (b)



For solid sphere moment of inertia about diameter

$$I_{AB} = \frac{2}{5}MR^2 = I$$

For solid disc

$$I_{A'B'} = I_{YY'} + Mr^2 = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$$

$I_{AB} = I_{A'B'}$ (given)

$$\therefore \quad \frac{2}{5}MR^2 = \frac{3}{2}Mr^2 \Rightarrow r = \frac{2}{\sqrt{15}}R$$

2. (a)

3. (a) Mass of disc = 4M

$$\text{I disc} = \frac{1}{2}MR^2$$

$\therefore$ Moment of inertia of one quarter of disc

$$= \frac{1}{4}\left[\frac{1}{2}(4M)R^2\right] = \frac{1}{2}MR^2$$

4. (d) About the diameter of the circular loop (ring)

$$I = \frac{1}{2}MR^2$$

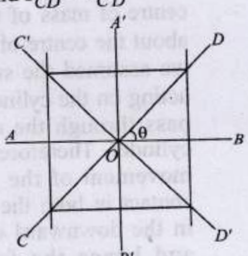
Using parallel axis theorem
Moment of inertia of the loop about XX' axis

$$I_{XX'} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

Here mass $M = L\rho$ and radius $R = \frac{L}{2\pi}$;

$$\therefore \quad I_{XX'} = \frac{3}{2}(L\rho)\left(\frac{L}{2\pi}\right)^2 = \frac{3L^3\rho}{8\pi^2}$$

5. (a) From symmetry $I_{AB} = I_{A'B'}$ and $I_{CD} = I_{C'D'}$
$A'B' \perp AB$ and $C'D' \perp CD$
From theorem of perpendicular axes,



$$I_{zz} = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$$

$$\Rightarrow \quad 2I_{AB} = 2I_{CD}$$

$$\therefore \quad I_{AB} = I_{CD} = I$$

6. (18) Linear impulse, $\int F dt = \Delta$ momentum $= m(V_{cm} - 0)$

or, $P = m(\omega r_{cm}) = m\omega\left(L + \frac{L}{2}\right)$

or, $P = m\omega\left(\frac{3L}{2}\right)$ ... (i)

And angular impulse $\int \tau dt = \Delta$ angular momentum

$\int r \times F dt = \Delta L \Rightarrow r \times \int F dt = I(\omega - 0)$ Here I = moment of inertia about axis of rotation.

$$\left(L + \frac{L}{2} + x\right) \times P = \left(I_{cm} + md^2\right)\omega$$

$$= \left(\frac{mL^2}{12} + m\left(L + \frac{L}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2} + x\right)P = mL^2\left(\frac{1}{12} + \left(\frac{3}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2} + x\right)P = mL^2\left(\frac{7}{3}\right)\omega \quad \text{... (ii)}$$

Dividing equation (ii) by (i)

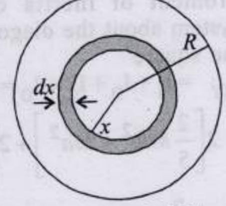
$$\left(\frac{3L}{2} + x\right) = \frac{L\left(\frac{7}{3}\right)}{\left(\frac{3}{2}\right)} \Rightarrow \frac{3L}{2} + x = L\left(\frac{14}{9}\right)$$

or, $x = \frac{L}{18} \quad \therefore \ n = 18$

7. (6) For solid sphere A, density

$$\rho_A(r) = k\left(\frac{r}{R}\right)$$

And $P_B(r) = K\left(\frac{r}{R}\right)^5$



Consider a spherical shell of radius x and thickness dx.
Mass of the shell, dm = density × volume

$$= \left(k\frac{x}{R}\right)(4\pi x^2\, dx)$$

So, moment of inertia of shell about its diameter,

$$dI = \frac{2}{3}(dm)x^2 = \frac{2}{3}\left(k\frac{x}{R}\right)(4\pi x^2 dx)x^2 = \left(\frac{8\pi}{3}\frac{k}{R}\right)x^5 dx$$

$\therefore$ Moment of inertia of the sphere A, $I_A = \int_0^R dI$

$$= \frac{8\pi k}{3R}\int_0^R x^5 dx = \frac{8\pi k}{3R}\left[\frac{x^6}{6}\right]_0^R$$

$$\therefore \quad I_A = \left(\frac{8\pi k}{18}\right)R^5 \quad \text{(i)}$$

Similarly, for sphere B

$$I_B = \frac{8\pi k}{3R^5}\int_0^R x^9 dx = \left(\frac{8\pi k}{3R^5}\right)\left[\frac{x^{10}}{10}\right]_0^R$$

$$\therefore \quad I_B = \frac{8\pi k}{30}R^5 \quad \text{(ii)}$$

From eqns. (i) and (ii)

$$\frac{I_B}{I_A} = \frac{18}{30} = \frac{6}{10} = \frac{n}{10} \quad \therefore n = 6$$

8. **(3)** Let σ be the surface mass density.
Moment of inertia of the lamina about axes passing through 'O'

$$I_O = \frac{1}{2}\sigma[\pi(2R)^2]\times(2R)^2 - \left[\frac{1}{2}(\sigma\pi R^2)^2 + \sigma(\pi R^2)\times R^2\right]$$

$$= \frac{13}{2}\pi\sigma R^4$$

Moment of inertia of the lamina about axes passing through 'P'

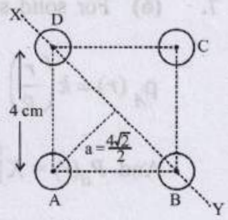
$$I_P = 8\pi\sigma R^4 + \sigma\pi(2R)^2 \times (2R)^2$$

$$\left[\frac{1}{2}\sigma(\pi R^2)R^2 + \sigma(\pi R^2)\left(\sqrt{(2R)^2 + R^2}\right)^2\right]$$

$$= 24\pi\sigma R^4 - 5.5\sigma\pi R^4 = 18.5\pi\sigma R^4$$

$$\therefore \frac{I_P}{I_O} = \frac{18.5\pi\sigma R^4}{\frac{13}{2}\pi\sigma R^4} = \frac{37}{13} \approx 3$$

9. **(9)** Let the four spheres be A, B, C, & D
$m_A = m_B = m_C = m_D = 0.5$ kg
Moment of inertia of the system about the diagonal of the square



$$I_{XY} = I_A + I_B + I_C + I_D = 2I_A + 2I_B$$

$$= 2\left[\frac{2}{5}MR^2 + Ma^2\right] + 2\left[\frac{2}{5}MR^2\right]$$

$$= 4\times\frac{2}{5}MR^2 + 2Ma^2 = M\left[\frac{8}{5}R^2 + 2(a)^2\right]$$

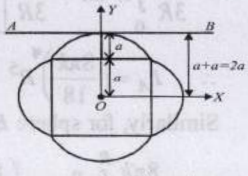
$$= 0.5\left[\frac{8}{5}\times\left(\frac{\sqrt{5}}{2}\right)^2 + 2\times 8\right]\times 10^{-4}$$

$$= 0.5[2+16]\times 10^{-4} = 9\times 10^{-4} = N\times 10^{-4}\text{ kg }m^2 \therefore N = 9$$

10. Assuming symmetric lamina to be in xy plane, $I_x = I_y$
By symmetry
$I_x + I_y = I_z$ (perpendicular-axis theorem)

$$I_x = I_y = \frac{I_z}{2} = 0.8Ma^2$$

$\because$ $I_z = 1.6\,Ma^2$ (given)
Now, from to parallel-axes theorem,

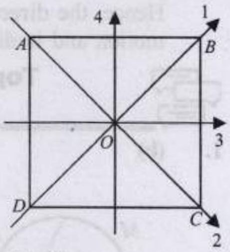


$$I_{AB} = I_x + M(2a)^2$$
$$= 0.8\,Ma^2 + 4Ma^2$$
$$= 4.8\,Ma^2$$

11. **(a, b, c)** Since, $ABCD$ is a square lamina hence by symmetry $I_1 = I_2$ and $I_3 = I_4$

From perpendicular axes theorem,

Moment of inertia about an axis perpendicular to square plate and passing from centre, O



$I_o = I_1 + I_2 = I_3 + I_4$
or $I_o = 2I_2 = 2I_3 \quad \therefore I_2 = I_3$
$I_1 = I_2 = I_3 = I_4$
Therefore, I_o can be obtained by adding any two
i.e., $I_o = I_1 + I_2 = I_1 + I_3$
$= I_1 + I_4 = I_2 + I_3$
$= I_2 + I_4 = I_3 + I_4$

Topic-5: Rolling Motion

1. **(b)** According to question, roller is moved 50 cm i.e., $V_{center} \cdot t = 50$ cm

Radius of roller, $R = \frac{20}{2} = 10$ cm and of axle, $r = \frac{10}{2} = 5$ cm

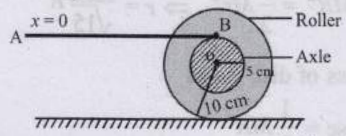
For no slipping at the ground

$V_{center} = \omega R \therefore$ Velocity of scale $= (V_{center} + \omega r)$

$\therefore$ Distance moved by scale $= (V_{center} + \omega r)t$

$$= \left(V_{center} + \frac{V_{center}r}{R}\right)t\left(\because \omega = \frac{V}{R}\right)$$

$$= \frac{3V_{center}}{2}\cdot t = \frac{3}{2}\times 50 = 75\text{ cm}$$



2. **(d)** Applying energy conservation principle of

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mg\left(\frac{3v^2}{4g}\right)$$

$$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}mv^2 \quad [\because v = R\omega]$$

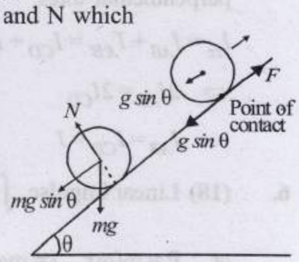
$$\therefore \quad \frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}mv^2 - \frac{1}{2}mv^2 = \frac{1}{4}mv^2 \Rightarrow I = \frac{1}{2}mR^2$$

This is the moment of inertia of the disc hence the object is disc.

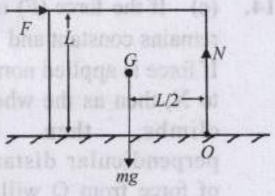
3. **(a)** In pure rolling, the point of contact is the instantaneous centre of rotation of all the particles of the disc. On applying $v = r\omega$
We find ω is same for all the particles then $v \propto r$.

$$r_Q > r_C > r_P \Rightarrow v_Q > v_C > v_P$$

4. **(b)** Cylinder to be moving on a frictionless surface. In both the cases roll up roll down the acceleration of the centre of mass of the cylinder is g sin θ. Also no torque about the centre of cylinder is acting on the cylinder since we assumed the surface to be frictionless and the forces acting on the cylinder is mg and N which pass through the centre of cylinder. Therefore the net movement of the point of contact in both the cases is in the downward direction and hence the frictional force always acts in the upward direction.



5. (c) Here, due to applied force normal reaction shifts to one corner. At this situation, the cubical block starts topping about point O.



About point O torque

$F \times L = mg \times \frac{L}{2} \Rightarrow F = \frac{mg}{2}$

Hence minimum force required to topple the block,

$F = \frac{mg}{2}$

6. (7) $K.E_{Total}$ of a rolling disc $K.E_{trans} + K.E_{rot}$

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right)$

$K.E_{Total} = \frac{3}{4}mv^2$

For surface AB

$k.E_i$ + loss in gravitational potential energy = $K.E_f$

$\frac{3}{4}m(3)^2 + mg(30) = \frac{3}{4}mV_B^2$ ...(i)

For surface CD

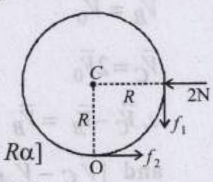
$\frac{3}{4}m(v_2)^2 + mg(27) = \frac{3}{4}mV_D^2$ ...(ii)

$\because V_B = V_D$. $\therefore$ from eq. (i) and (ii)

$\frac{3}{4}m(3)^2 + mg \times 30 = \frac{3}{4}m(v_2)^2 + mg \times 27$

$\therefore V_2 = 7\ m/s$

7. (4) The stick applies (2N) force so, point of contact O of the ring with ground tends to slide. But the frictional force f_2 does not allow this and creates a torque about 'c' which starts rolling the ring. Between the ring & the stick a friction force f_1 also acts.



$F - f_2 = ma$

$2 - f_2 = 2 \times 0.3 \quad \therefore f_2 = 1.4$ N

Applying $\tau = I\alpha$ about C

$(f_2 - f_1)R = I\alpha = I\frac{a}{R} \quad [\because a = R\alpha]$

$\Rightarrow [1.4 - \mu \times 2] \times 0.5 = 2 \times (0.5)^2 \times \frac{0.3}{0.5} \quad [\because I = MR^2]$

$\therefore \mu = 0.4 = \frac{4}{10} = \frac{P}{10} \quad \therefore P = 4$

8. (0.52) The angle rotated by disc in $t = \sqrt{\pi}$ sec is

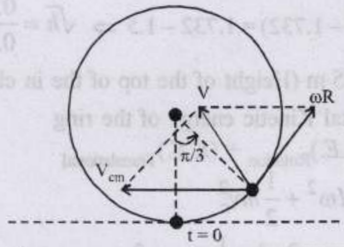
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times \frac{2}{3}\left(\sqrt{\pi}\right)^2 = \frac{\pi}{3}$ rad

Angular velocity at the end of $t = \sqrt{\pi}$ sec is

$\omega = \omega_0 + \alpha t = \frac{2}{3}\sqrt{\pi}$ rad / sec

and, as it is pure rolling so $V_{cm} = \omega R = \frac{2}{3}\sqrt{\pi} \times 1$ m/s

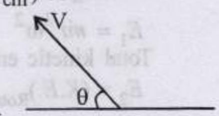
So, at the moment of detachment we get the following situation



So, $V = \sqrt{V_{cm}^2 + (\omega R)^2 + 2(\omega R)(V_{cm})\cos 120°}$

$= V_{cm} \quad [\because V_{cm} = \omega R]$

$= \frac{2\sqrt{\pi}}{3}$ m/s



$\tan\theta = \frac{(\omega R)\sin 120°}{V_{cm} + (\omega R)\cos 120°} = \frac{\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}} = \sqrt{3}$

So, $\theta = 60°$

Then, $H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{\left(\frac{2\sqrt{\pi}}{3}\right)^2 \times \frac{3}{4}}{2 \times 10} = \frac{\frac{4\pi}{9} \times \frac{3}{4}}{20} = \frac{\pi}{60}$ m

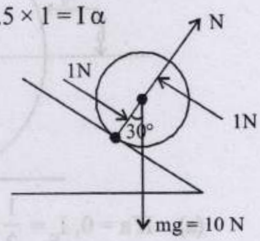
Height from ground $= R\left(1 - \cos\frac{\pi}{3}\right) + \frac{\pi}{60} = \frac{R}{2} + \frac{\pi}{60}$

$= \frac{1}{2} + \frac{\pi}{60} = \frac{1}{2} + \frac{1}{10}\left(\frac{\pi}{6}\right)$

So, $x = \frac{\pi}{6} \simeq 0.52$

9. (2.86) We know that for a rolling body, point of contact is instantaneous axis of rotation.

So, $\tau_{\text{point of contact}} = I\alpha$



$\Rightarrow 1 \times 0.5 + (mg\sin 30°)R - 1.5 \times 1 = I\alpha$

$\Rightarrow +0.5 + 5R - 1.5 = I\alpha$

$\Rightarrow 5R - 1 = \frac{7}{5}mR^2\alpha$

$\Rightarrow 4 = \frac{7}{5}\alpha$

$\Rightarrow \alpha = \frac{20}{7}$ rad / sec^2

So, $a_{cm} = \alpha R = \frac{20}{7} \times 1 = \frac{20}{7}$ m/s^2 = 2.86 m/s^2

10. (0.75) The time taken to reach the ground or descend

$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1 + \frac{K^2}{R^2}\right)}$

For ring $t_1 = \frac{1}{\sin 60°}\sqrt{\frac{2h}{g}(1+1)} = \frac{4}{\sqrt{3}}\sqrt{\frac{h}{g}} \quad \because \frac{K^2}{R^2} = 1$

For disc $t_2 = \frac{1}{\sin 60°}\sqrt{\frac{2h}{g}\left(1 + \frac{1}{2}\right)} = \sqrt{\frac{4h}{g}} \quad \because \frac{K^2}{R^2} = \frac{1}{2}$

Given $t_1 - t_2 = \frac{2 - \sqrt{3}}{\sqrt{10}}$

$\therefore \frac{4}{\sqrt{3}}\sqrt{\frac{h}{g}} - \sqrt{\frac{4h}{g}} = \frac{2 - \sqrt{3}}{\sqrt{10}} \Rightarrow 2\sqrt{h} - \sqrt{3h} = \sqrt{3} - \frac{3}{2}$

$\Rightarrow \sqrt{h}\,(2-1.732) = 1.732 - 1.5 \Rightarrow \sqrt{h} = \dfrac{0.232}{0.268}$

$\therefore h \approx 0.75$ m (Height of the top of the in clined plane)

11. **False.** Total Kinetic energy of the ring

$E_1 = (K.E.)_{Rotation} + (K.E.)_{Translational}$

$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_c^2$

$= \frac{1}{2}\times mr^2\omega^2 + \frac{1}{2}m(r\omega)^2 \quad (\because I = mr^2,\ v_c = r\omega)$

$E_1 = mr^2\omega^2$

Total kinetic energy of the cylinder

$E_2 = (K.E.)_{Rotation} + (K.E.)_{Translational}$

$= \frac{1}{2}I'\omega'^2 + \frac{1}{2}Mv'^2_c = \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega'^2 + \frac{1}{2}M(r\omega')^2$

$E_2 = \frac{3}{4}Mr^2\omega'^2$

Equating $E_1 = E_2$

$mr^2\omega^2 = \frac{3}{4}Mr^2\omega'^2 \Rightarrow \frac{\omega'^2}{\omega^2} = \frac{4m}{3M} = \frac{4}{3}\times\frac{0.3}{0.4} = 1 \Rightarrow \omega' = \omega$

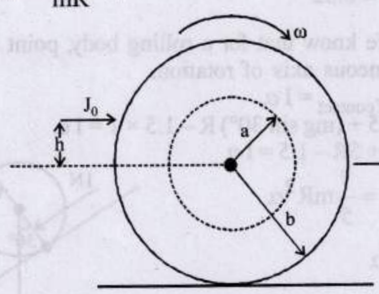
Hence both ring and cylinder will reach at the same time.

12. **(a, b, c, d)** Impulse $\vec{J} = \Delta\vec{P}$ so

$J_0 = mv$ .....(i)

$J_0h_m = I_c\omega \Rightarrow h_m = \frac{I_c\omega}{J_0} = \frac{I_c\omega}{mv}$

$\Rightarrow h_m = \frac{I_c}{mR} \quad [\because V = R\omega]$



(a) If $a = 0$, $I_c = \frac{1}{2}mb^2$ & $R = b \therefore h_m = \frac{b}{2}$

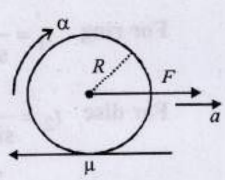
(b) If $a = b$, $I_C = mb^2$ & $R = b \therefore h_m = b$

(c) $v = \frac{J_0}{m} \Rightarrow 100 = \frac{V}{R} = \frac{J_0}{mR}$

(d) As force is acting on centre of mass so no rotation for $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

13. **(b, d)** For solid cylinder,

$F\times R = \frac{3}{2}mR^3\times\left(\frac{a}{R}\right) \Rightarrow a = \frac{2F}{m}$



For hollow cylinder,

$F\times R = (2mR^2)\times\frac{a}{R} \Rightarrow a = \frac{F}{2m}$

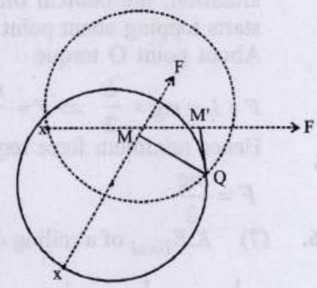
For solid cylinder

$f = F - m\times\frac{2F}{3m} = \frac{F}{3} \le \mu mg \Rightarrow F \le 3\mu mg$

$\therefore a_{max} = 2\mu g$

14. **(c)** If the force (F) is applied at P tangential than the τ remains constant and $\tau = F\times 2R$.

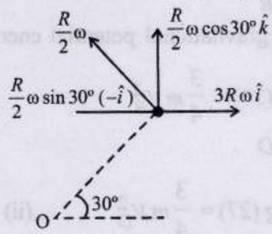
If force is applied normal to X, then as the wheels climbs, then the perpendicular distance of force from Q will go on changing initially the perpendicular is QM, later it becomes QM'.



If the force (F) is applied normal to the circumference at point P then τ is zero.

If the force (F) is applied tangentially at point S then $\tau = F\times R$ and the wheel climbs.

15. **(a, b)** Velocity at centre 'O' $\therefore\ \vec{v}_0 = 3R\omega\,\hat{i}$



$\vec{V}_P = 3R\omega\hat{i} - \frac{R\omega}{2}\sin 30°\hat{i} + \frac{R\omega}{2}\cos 30°\hat{k}$

$\therefore \quad \vec{V}_P = \left[3R_\omega\,\hat{i} - \frac{R_\omega}{4}\hat{i}\right] + \frac{\sqrt{3}R_\omega}{4}\hat{k}$

or, $\vec{V}_P = \frac{11}{4}R_\omega\,\hat{i} + \frac{\sqrt{3}}{4}R_\omega\,\hat{k}$

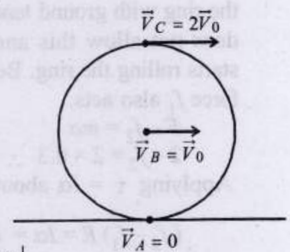
16. **(b, c)** Here, $\vec{V}_A = 0$

$\vec{V}_B = \vec{V}_0$

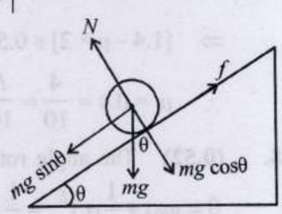
$\vec{V}_C = 2\vec{V}_0$

$\therefore \vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

and $|\vec{V}_C - \vec{V}_A| = |\vec{V}_B - \vec{V}_C|$



17. **(c, d)** The sliding friction facts in the opposite direction of mg sin θ to oppose the relative motion. Because of frictional force the cylinder rolls.



Hence frictional force aids rotation but hinders translational motion.

Applying $F_{net} = ma$ along the direction of inclined plane,

$mg\sin\theta - f = ma_c$,

where a_c = acceleration of centre of mass of the cylinder

$\therefore \quad f = mg\sin\theta - ma_c$ ... (i)

But $a_c = \dfrac{g\sin\theta}{1+\dfrac{I_c}{mR^2}} = \dfrac{g\sin\theta}{1+\dfrac{mR^2/2}{mR^2}} = \frac{2}{3}g\sin\theta$

$f = \dfrac{mg\sin\theta}{3}$

Clearly, if θ is reduced, frictional force is reduced.

18. (d) In case of pure rolling on inclined plane acceleration of a body

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

For hollow cylinder $\frac{I}{MR^2} = \frac{MR^2}{MR^2} = 1$

For solid cylinder $\frac{I}{MR^2} = \frac{\frac{1}{2}MR^2}{MR^2} = \frac{1}{2}$

Hence acceleration of solid cylinder is more than hollow cylinder and therefore solid cylinder will reach the bottom of the inclined plane first.

In the case of rolling there will be no work done by friction. Therefore total mechanical energy remains conserved.

$\therefore$ $(KE)_{solid} = (KE)_{hollow}$ = decrease in PE = mgh

19. The man applies a force. Its horizontal component F pushes the plank. Hence the point of contact of the plank with the cylinder will tend to move towards right the frictional force f_1 will act towards left on the plank.

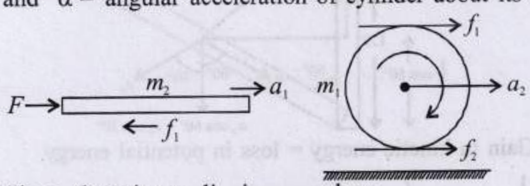
Let f_1 = friction between plank and cylinder

f_2 = friction between cylinder and ground

a_1 = acceleration of plank

a_2 = acceleration of centre of mass of cylinder

and α = angular acceleration of cylinder about its CM.



Since, there is no slipping anywhere

$\therefore$ $a_1 = 2a_2$ ... (i)

$$a_1 = \frac{F - f_1}{m_2} \quad \text{... (ii)}$$

$$a_2 = \frac{f_1 - f_2}{m_1} \quad \text{... (iii)}$$

$$\alpha = \frac{(f_1 - f_2)R}{I} = \frac{(f_1 - f_2)R}{\frac{1}{2}m_1R^2}$$

$$\alpha = \frac{2(f_1 - f_2)}{m_1R} \quad \text{... (iv)}$$

$$a_2 = R\alpha = \frac{2(f_1 - f_2)}{m_1} \quad \text{... (v)}$$

(a) Solving eqs. (i) to (v), we get

Acceleration of plank, $a_1 = \frac{8F}{3m_1 + 3m_2}$

And Acceleration of centre of mass of cylinder,

$$a_2 = \frac{4F}{3m_1 + 8m_2}$$

(b) Friction between plank and cylinder $f_1 = \frac{3m_1F}{3m_1 + 8m_2}$

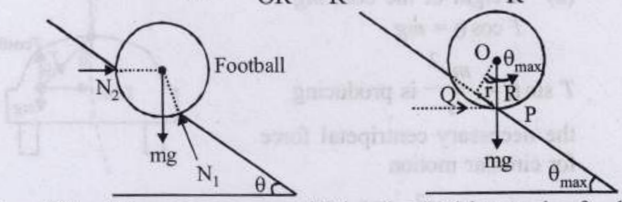
Friction between cylinder and ground $f_2 = \frac{m_1F}{3m_1 + 8m_2}$

Since, all quantities are positive, they are correctly shown in figures.

Topic-6: Miscellaneous (Mixed Concepts) Problems

1. (a) The maximum value of q i.e, q_{max}, the football is about to roll, then $N_2 = 0$ and all the forces (mg and N_1) must pass through contact point 'P'

$\therefore \cos(90° - \theta_{max}) = \frac{OQ}{OR} = \frac{r}{R}$ or, $\sin\theta_{max} = \frac{r}{R}$



2. (b) Angular momentum, $|\mathbf{L}|$ or $L = I\omega$ (about axis of rod)

Moment of inertia of the rod-insect system.

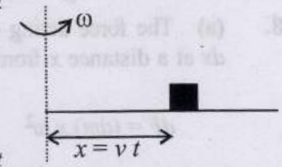
$I = I_{rod} + mx^2 = I_{rod} + mv^2t^2$

Here, m = mass of insect

$\therefore$ $L = (I_{rod} + mv^2t^2)\omega$

Now $|\tau| = \frac{dL}{dt}$

$= (2mv^2t\omega)$ or $|\tau| \propto t$



i.e. the graph is straight line passing through origin.

After time T, L = constant

$\therefore$ $|\tau|$ or $\frac{dL}{dt} = 0$

i.e., when the insect stops moving, L does not change and therefore T becomes constant.

3. (a) When we are giving an angular acceleration (α) to the rod, the bead has instantaneous acceleration crinst $= L\alpha$. The bead has a tendency to move away from the centre. But due to the friction between the bead and the rod, this does not happen. If instantaneous angular velocity is ω then

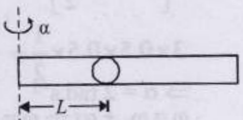
Here, necessary frictional force is provided by frictional force

$mL\omega^2 = \mu(ma) \Rightarrow mL\omega^2 = \mu\, mL\alpha \Rightarrow \omega^2 = \mu\alpha$

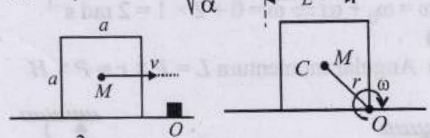
Using $\omega = \omega_0 + \alpha t$,

$\Rightarrow$ $\omega = \alpha t$

$\therefore$ $\alpha^2t^2 = \mu\alpha \Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$



4. (a)



$r^2 = \frac{a^2}{2}$ or $r = \sqrt{2}\frac{a}{2}$ By geometry

Net torque about point O is zero.

Hence, angular momentum (L) about O is conserved, or $L_i = L_f$

$$MV\left(\frac{a}{2}\right) = I_0\omega = (I_{cm} + Mr^2)\omega$$

$$\omega = \left\{\frac{Ma^2}{6} + M\left(\frac{a^2}{2}\right)\right\}\omega = \frac{2}{3}Ma^2\omega = \frac{3v}{4a}$$

5. (c) As it is headon collision between two identical spheres, they will exchange their linear velocities. Since the spheres are smooth there will be no friction (no torque) and therefore there will be no transfer of angular momentum.

Thus A, after collision will remain with its initial angular momentum. i.e., $\omega_A = \omega$ and $\omega_B = 0$

6. (c)

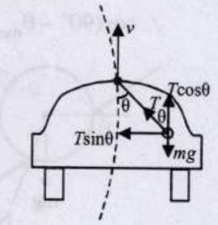
7. (c) The bob is undergoing a circular motion as the car is moving in circular horizontal track with a constant speed. The bob is under the influence of two forces.

(i) Tension, T in the rod

(ii) Weight of the bob, mg

$T\cos\theta = mg$

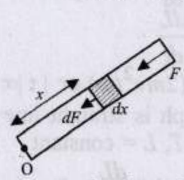
$T\sin\theta = \frac{mv^2}{r}$ is producing the necessary centripetal force for circular motion



$\therefore \quad \tan\theta = \frac{v^2}{rg} = \frac{10\times 10}{10\times 10} = 1 \quad \text{or } \theta = 45^\circ$

8. (a) The force acting on the mass of liquid dm of length dx at a distance x from the axis of rotation O.

$dF = (dm)\, x\, \omega^2 \qquad \left(\because F = \frac{mv^2}{r}\right)$



$\therefore \quad dF = \frac{M}{L} dx \times x\omega^2$

where $\frac{M}{L}$ is linear mass density of liquid.

$\therefore$ The force acting at the other end is for the whole liquid in tube.

$$F = \int_0^L \frac{M}{L}\omega^2 x\,dx = \frac{M}{L}\omega^2 \int_0^L x\,dx$$

$$= \frac{M}{L}\omega^2\left[\frac{x^2}{2}\right]_0^L = \frac{M}{L}\omega^2\left[\frac{L^2}{2} - 0\right] = \frac{ML\omega^2}{2}$$

9. (2) $3\left[F\times r\times\frac{1}{2}\right] = I\alpha$

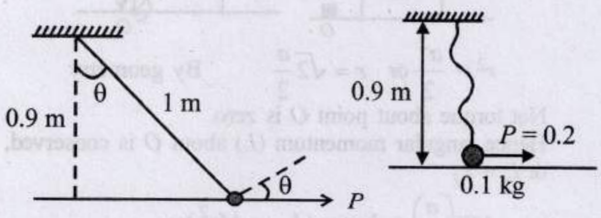
$3\times 0.5\times 0.5\times\frac{1}{2} = \frac{1}{2}\times 1.5\times 0.5\times 0.5\times\alpha$

$\Rightarrow \alpha = 2 \text{ rad s}^{-1}$

$\omega = \omega_0 + \alpha t \Rightarrow \omega = 0 + 2\times 1 = 2 \text{ rad s}^{-1}$

10. (0.18)

11. (0.16) Angular momentum $L = P\times r = P\times H$



or, $L = 0.2\times 0.9 = 0.18 \text{ kgm}^2/\text{s}$

$\therefore \quad J = \mathbf{0.18}$

There will be no velocity along the string just after string becomes taut.

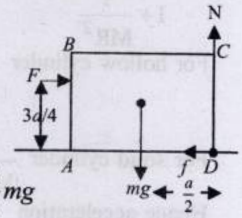
$\therefore \quad V_\perp = \frac{P\cos\theta}{m} = \frac{0.2\times 0.9}{1\times 0.1} = 1.8 \text{ m/s}$

$\therefore \quad$ Kinetic energy, $K = \frac{1}{2}mv_\perp^2 = \frac{1}{2}\times 0.1\times(1.8)^2 = \mathbf{0.162\ J}$

12. When the cube begins to tip about the edge the normal reaction N will pass through the edge D about which rotation takes place. The moments due to N and frictional force f will be zero.

Taking moment of force F and mg about D

$F\times\frac{3a}{4} = mg\times\frac{a}{2} \therefore F = \frac{2}{3}mg$

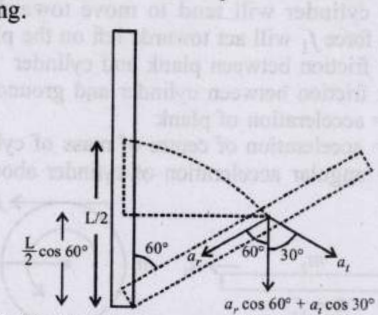


Hence, minimum value of $F = \frac{2}{3}mg$

13. False torque, $\tau = I\alpha \quad \therefore \alpha = \frac{\tau}{I}$

$\tau = F\times r\perp$ Torque is same in both the cases. But since, I will be different due to different mass distribution about the axis, so α will be different.

14. (a, b, c) The rod is released from rest so that it falls by rotating about its contact point with the floor without slipping.



Gain in kinetic energy = loss in potential energy

$\frac{1}{2}I\omega^2 = mg\frac{l}{2}(1-\cos 60^\circ)$

$\therefore \quad \frac{ml^2}{3}\omega^2 = mg\frac{l}{2} \quad \Rightarrow \quad \omega = \sqrt{\frac{3g}{2l}}$

Now, $\tau = I\alpha$

$\therefore \quad mg\times\frac{l}{2}\sin 60^\circ = \frac{1}{3}ml^2\alpha \quad \Rightarrow \alpha = \frac{3\sqrt{3}}{4}\frac{g}{l}$

Further $a_t = \frac{l}{2}\alpha = \frac{3\sqrt{3}g}{8}$

Also $a_r = \omega^2\frac{l}{2} = \frac{3g}{2l}\times\frac{l}{2} = \frac{3g}{4}$

For vertical motion of centre of mass

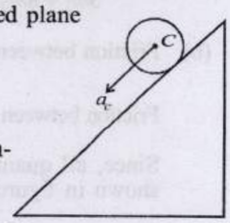
$mg - N = m(a_r\cos 60^\circ + a_t\cos 30^\circ)$

$\therefore \quad mg - N = m\left[\frac{3g}{4}\times\frac{1}{2} + \frac{3\sqrt{3}g}{8}\times\frac{\sqrt{3}}{2}\right]$

$\therefore \quad N = \frac{Mg}{16}$

15. (d) As we know, acceleration of the center of mass of cylinder rolling down an inclined plane

$$a_c = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

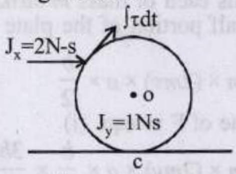


In case of P the mass is concentrated away from the axis,

So $I_P > I_Q$

$\therefore \quad a_P < a_Q \Rightarrow v_P < v_Q \Rightarrow \omega_P < \omega_Q$

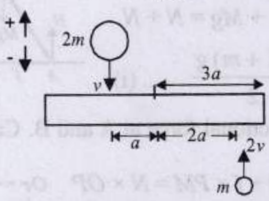
16. **(c)** The angular impulse created by the frictional force between the ring and the ball tends to decrease the angular speed ω of the ring about O.



After the collision ω decreases but the ring remains rotating in the anticlockwise direction. Hence the friction between the ring and the ground at the point of contact is to the left.

17. **(a, c, d)** Applying conservation of linear momentum i.e., $P_i = P_f$

$2m(-v) + m(2v) + 8m \times 0 = (2m + m + 8m)\,v_c \Rightarrow v_c = 0$



Now, applying conservation of angular momentum about centre of mass i.e., $L_i = L_f$

$2mv \times a + m(2v) \times 2a = I\omega$

Here, $I = \frac{1}{12}(8m) \times (6a)^2 + 2m \times a^2 + m \times 4a^2 = 30ma^2$

$\therefore \quad 2mv(a) + m(2v) \times 2a = 30ma^2 \times \omega$

$\Rightarrow \omega = \frac{v}{5a}$

$\therefore$ After collision energy, $E = \frac{1}{2} I\omega^2$

$= \frac{1}{2} \times 30\,ma^2 \times \frac{v^2}{25a^2} = \frac{3mv^2}{5}$

18. **(a)**

19. **A → (p,t) ; B → (q,s,t) ; C → (p,r,t) D → (q, p)**

As the velocity is constant

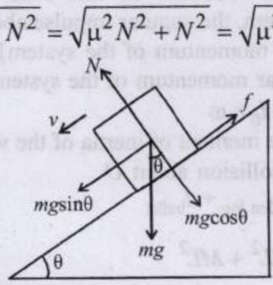
$\therefore \quad f = mg\sin\theta$

But $f = \mu N = \mu mg\cos\theta$

$\therefore \quad \mu mg\cos\theta = mg\sin\theta \Rightarrow \mu = \tan\theta$

The force by X on Y is the resultant of f and N

$\sqrt{f^2 + N^2} = \sqrt{\mu^2 N^2 + N^2} = \sqrt{\mu^2 + 1}N$

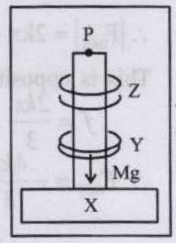


$= (\sqrt{\tan^2\theta + 1})\, mg\cos\theta = \sec\theta mg\cos\theta = mg$

= weight of Y.

Again, due to the presence of frictional force between Y and X, the mechanical energy of the system $(X + Y)$ decreases continously as Y slides down.

(q) Lift moves up, X also moves up and therefore the gravitational energy of X is continuously increasing.

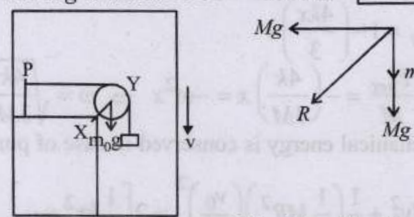


T of weight of Y about P as the perpendicular distance of the line of action of force from the point P is zero.

Force exerted by X on Y

$= Mg + Mg = 2mg$ where Mg is wt. of Y and Mg is the force on Y due to Z.

(r)



In this case the force exerted by X on Y = force exerted by Y on X. The force on X due to Y is

$$R = \sqrt{(Mg)^2 + [(m_0 + M)g]^2} \neq Mg$$

The mechanical energy of the system ($X + Y$) is continously decreasing as the system is coming down and its potential energy is decreasing, the kinetic energy remaining the same.

The torque of the weight of Y about $P \neq 0$

(s) Force on Y by X is = wt. of liquid displaced which cannot be equal to Mg as the density of Y > density of X (∵ Y is sinking)

The gravitational potential energy of X increases continously because as Y moves down, the centre of mass of X moves up.

(t) Sphere Y is moving with terminal velocity V_T.

$\therefore$ Net force on Y is zero i.e. $Mg = B + F_v$

$B + F_v$ are exerted by X on Y.

The gravitational potential energy of X is continously increasing because as Y moves down, the centre of mass of X moves up.

The mechanical energy of the system ($X + Y$) is continously decreasing to overcome the viscous forces.

20. **(c)** Here $\omega_0(R - r) = \omega R \quad \therefore \quad \omega = \omega_0\left(\frac{R - r}{R}\right)$

Total kinetic energy of the ring = (Kinetic rotational + kinetic energy translational)

$$K.E_{total} = \frac{1}{2}(2MR^2)\omega^2 = M\omega_0^{\,2}(R - r)^2$$

21. **(a)** $\mu M\omega_{min}^2 (R - r) = Mg$

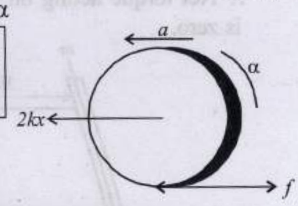
$\therefore \omega_{min} = \sqrt{\frac{g}{\mu(R - r)}}$

22. **(a)** Axis of rotation is parallel to z-axis. Hence for both the cases, in stantaneous axis passing through is vertical.

23. **(d)** For rigid body ω is same for any point of the body.

24. **(d)** $2kx - f = Ma$ and, $a = R\alpha$

$\therefore \quad \frac{2kx - f}{M} = R\left[\frac{fR}{\frac{1}{2}MR^2}\right]$



$= R\left[\frac{fR}{\frac{1}{2}MR^2}\right]$

Solving this equation, we get

$\therefore |F_{net}| = 2kx - f = 2kx - \frac{2kx}{3} = \frac{4kx}{3}$

This is opposite to displacement

$f = \frac{2kx}{3}$

$\therefore F_{net} = -\frac{4kx}{3}$ directed towards the equilibrium

25. (d) $F_{net} = 1 - \left(\frac{4kx}{3}\right)x$

$\therefore \quad a = \frac{F_{net}}{M} = -\left(\frac{4k}{3M}\right)x = -\omega^2 x \Rightarrow \omega = \sqrt{\frac{4k}{3M}}$

26. (c) Mechanical energy is conserved in case of pure rolling motion

$\therefore \frac{1}{2}Mv_0^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_0}{R}\right)^2 = 2\left[\frac{1}{2}kx^2{}_{max}\right]$

$\therefore \quad x_{max} = \sqrt{\frac{3M}{4k}}v_0$

$F_{max} = \mu Mg = \frac{2kx_{max}}{3} = \frac{2k}{3}\sqrt{\frac{3M}{4k}}v_0$

$\therefore \quad v_0 = \mu g\sqrt{\frac{3M}{k}}$

27. (b) Loss in kinetic energy $= (K.E.)_{initial} - (K.E.)_{final}$

$= \left[\frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)\omega^2\right] - \left[\frac{1}{2}(I+2I)\left(\frac{4}{3}\omega\right)^2\right]$

$= 3I\omega^2 - \frac{8}{3}I\omega^2 = \frac{I\omega^2}{3}$

28. (a) When disc B is brought in contact with disc A

Let ω the common velocity. From conservation of angular momentum for the two disc system.

$I(2\omega) + 2I(\omega) = (I + 2I)\omega' \Rightarrow \omega' = \frac{4}{3}\omega$

Torque on disc A

$\tau_A = \frac{\Delta L_A}{t} = \frac{L_f - L_i}{t} = \frac{I \times \frac{4}{3}\omega - I \times 2\omega}{t} = \frac{-2I\omega}{3t}$

Here negative sign indicates that the torque creates angular retardation.

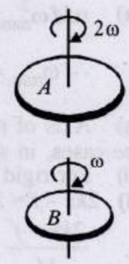
29. (c) For disc A

$\frac{1}{2}kx_1^2 = \frac{1}{2}I(2\omega)^2 \Rightarrow kx_1^2 = 2I\omega^2$

For disc B

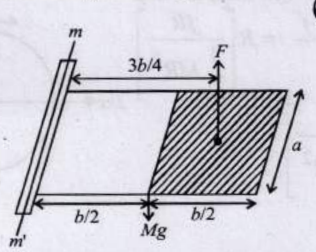
$\frac{1}{2}kx_2^2 = \frac{1}{2} \times 2I\omega^2 \Rightarrow kx_2^2 = I\omega^2$

$\therefore \frac{kx_1^2}{kx_2^2} = \frac{2I\omega^2}{I\omega^2} \Rightarrow \frac{x_1}{x_2} = \sqrt{2}$



30. Plate is held in horizontal position

$\therefore$ Net torque acting on the plate is zero.



$Mg \times \frac{b}{2} = F \times \frac{3b}{4}$ ... (i)

n number of balls each of mass m striking with velocity v to the shaded half portion of the plate

$F = n\frac{dp}{dt} \times A = n \times (2mv) \times a \times \frac{b}{2}$ $\quad (\because A = \text{area})$

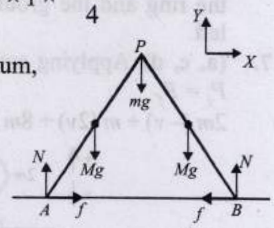
Putting this value of F in eqn. (i)

$Mg \times \frac{b}{2} = n \times (2mv) \times a \times \frac{b}{2} \times \frac{3b}{4}$

$\Rightarrow 3 \times 10 = 100 \times 2 \times 0.01 \times v \times 1 \times \frac{3 \times 2}{4}$

$\therefore \quad v = 10$ m/s

31. Since the system is in equilibrium,



$\therefore \quad \Sigma F_y = 0$

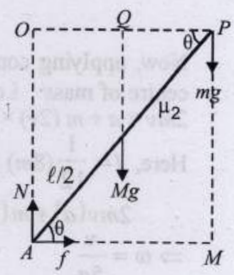
$Mg + mg + Mg - N - N = 0$

$\Rightarrow \quad Mg + mg + Mg = N + N$

$\Rightarrow \quad N = \frac{(2M+m)g}{2}$ ... (i)

Let f be the frictional force at A and B. Calculating torques about P

$Mg \times PQ + f \times PM = N \times OP$



$\Rightarrow \quad Mg \times \frac{L}{2}\cos\theta + f \times L\sin\theta$

$= NL\cos\theta$

$\Rightarrow \quad f = \frac{NL\cos\theta - \frac{MgL}{2}\cos\theta}{L\sin\theta}$

$= N\cot\theta - \frac{Mg}{2}\cot\theta$

$\Rightarrow \quad f = \left[\left(\frac{2N+m}{2}\right)g - \frac{Mg}{2}\right]\cot\theta$

$\Rightarrow \quad f = \left[(M+m)\frac{g}{2}\right]\cot\theta$

32. A bullet of mass m strikes the wooden log of mass M and length L and sticks to it.

Torque $\vec{\tau} = \frac{d\vec{L}}{dt} \quad \Rightarrow \vec{\tau} \times dt = d\vec{L}$

When angular impulse $(\vec{\tau} \times dt)$ is zero, the angular momentum is constant. In this case for the wooden log-bullet system, the angular impulse about O is constant.

$\therefore$ [angular momentum of the system]$_{initial}$

= [angular momentum of the system]$_{final}$

$\Rightarrow \quad mv \times L = I_0 \times \omega$ ...(i)

where I_0 is the moment of inertia of the wooden log-bullet system after collision about O

$I_0 = I_{wooden\ log} + I_{bullet}$

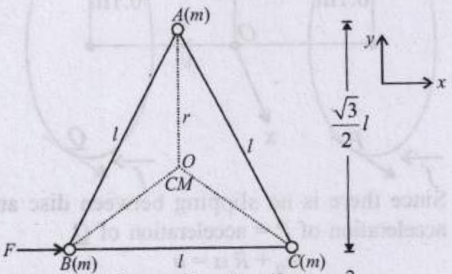
$= \frac{1}{3}ML^2 + ML^2$ ...(ii)

From eq. (i) and (ii)

$\omega = \frac{mv \times L}{\left[\frac{1}{3}ML^2 + mL^2\right]} = \frac{3mv}{(M+3m)L}$

33. (a) Let point 'O' denote the centre of mass (CM) of the system

$\therefore \quad OA = r = \dfrac{l}{\sqrt{3}}$ (By geometry)



F = centripetal force of $\quad F = (3m)\, r\omega^2$

or $\quad F = (3m)\left(\dfrac{1}{\sqrt{3}}\right)\omega^2$

or $\quad F = \sqrt{3}ml\omega^2$

(b) Let α be the angular acceleration of system about point A

$$\alpha = \frac{\tau_A}{I_A} = \frac{(F)\left(\frac{\sqrt{3}}{2}l\right)}{2ml^2} = \frac{\sqrt{3}F}{4ml}$$

Now, acceleration of CM along x-axis is

$$a_x = r\alpha = \left(\frac{l}{\sqrt{3}}\right)\left(\frac{\sqrt{3}F}{4ml}\right) \quad \text{or} \quad a_x = \frac{F}{4m}$$

Let F_x be the force applied by the hinge along X-axis.

$\therefore \quad F_x + F = (3m)a_x$

or $\quad F_x + F = (3m)\left(\dfrac{F}{4m}\right) \quad$ or $\quad F_x + F = \dfrac{3}{4}F$

or $\quad F_x = \dfrac{F}{4}$

Let F_y be the force applied by the hinge along Y-axis

$\therefore \quad F_y$ = centripetal force

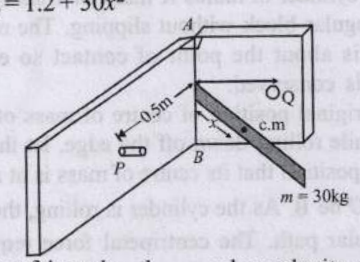
or $\quad F_y = \sqrt{3}ml\omega^2$

34. Let x be the distance of centre of mass from line AB.

$\therefore$ M.I. of laminar sheet about AB

$I_{AB} = I_{c.m.} + mx^2$ (From parallel axes theorem)

$I_{AB} = 1.2 + 30x^2$ ... (i)



Because of impulse the angular velocity of the laminar sheet will change after every impact.

Impulse = change in linear momentum

$6 = 30\,(V_f - V_i)$

$6 = 30 \times x\,(\omega_f - \omega_i)$ ... (ii)

Also, change in angular momentum = moment of impulse

$\therefore \quad I_{AB}\omega_f - I_{AB}\omega_i$ = Impulse × distance

$I_{AB}(\omega_f - \omega_i) = 6 \times 0.5 = 3$

$\therefore \quad \omega_f = \dfrac{3}{I_{AB}} + \omega_i = \dfrac{3}{1.2 + 30x^2} + (-1)$ ... (iii)

From eq. (ii) and (iii)

$$6 = 30 \times x\left[\frac{3}{1.2 + 30x^2} - 1 + 1\right] \quad (\because \omega_i = 1 \text{ rod/s})$$

$$1 = 5x\left[\frac{3}{1.2 + 30x^2}\right] \Rightarrow 1.2 + 30x^2 = 5x\,[+3] = 15x$$

$\therefore \quad 30x^2 - 15x - 1.2 = 0$

Solving, we get $x = 0.1$ or 0.4

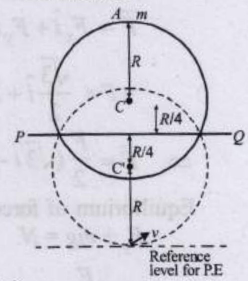
But at $x = 0.4$ m ω_f comes out to be negative (– 0.5 rod/s) which is not acceptable. Therefore,

(a) x = distance of CM from line AB = 0.1 m

(b) Substituting $x = 0.1$ m in eq. (ii) we get $\omega_f = 1$ rod/s the angular velocity with which sheet comes back after the first impact is 1 rad/s.

(c) As the sheet returns with same angular velocity of 1 rad/s, the sheet will never come to rest.

35. Initially the disc is held vertical with the point A at its highest position. It starts rotating when allowed to fall. Hence its potential energy changes to rotational kinetic energy.



Applying energy conservation

Enitial energy = Final energy

$$mg\left(2R + \frac{2R}{4}\right) + mg\left(R + \frac{2R}{4}\right) = mgR + \frac{1}{2}I\omega^2$$

Where I = M.I. of disc-mass system about PQ

$$mg \times \frac{10R}{4} + mg\frac{6R}{4} = mgR + \frac{1}{2}I\omega^2 \Rightarrow 3mgR = \frac{1}{2}I\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{6mgR}{I}} \quad \text{... (i)}$$

$$(I)_{PQ} = (I_{disc})_{PQ} + (I_{mass})_{PQ}$$

$$= \left[\frac{mR^2}{4} + M\left(\frac{R}{4}\right)^2\right] + m\left(\frac{5R}{4}\right)^2$$

$\left[\because \text{M.I. of disc about diameter} = \dfrac{1}{4}MR^2\right]$

$$I = \frac{mR^2[4 + 1 + 25]}{16} = \frac{15mR^2}{8}$$

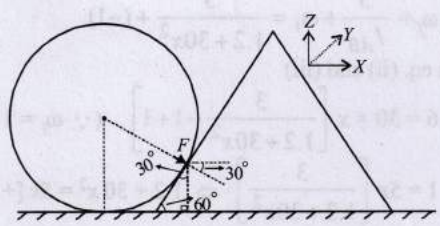
Putting this value of I in eq. (i)

$$\omega = \sqrt{\frac{6mgR \times 8}{15mR^2}} = \sqrt{\frac{16g}{5R}}$$

The linear speed of the particle as it reaches its lowest position

$$v = \omega\left(R + \frac{R}{4}\right) \quad \therefore \quad v = \sqrt{\frac{16g}{5R}} \times \frac{5R}{4} = \sqrt{5gR}$$

36. (a) The collision is elastic and the sphere is fixed, hence the wedge will return back with the same velocity (in magnitude).



The force responsible to change the velocity of the wedge in X-direction is F_x.

$$F_x \times \Delta t = mv - (-mv)$$

(Impulse) = (Change in momentum)

$$\therefore\ F_x = \frac{2mv}{\Delta t} \Rightarrow F\cos 30^\circ = \frac{2mv}{\Delta t} \Rightarrow F = \frac{4mv}{\sqrt{3}\,\Delta t}$$

$\because\ F_x = F\cos 30^\circ$ and $F_y = F\sin 30^\circ$

In vector terms

$$\vec{F} = F_x\hat{i} + F_y(-\hat{k}) = F\cos 30^\circ\hat{i} + F\sin 30^\circ(-\hat{k})$$

$$= F \times \frac{\sqrt{3}}{2}\hat{i} + F \times \frac{1}{2}(-\hat{k})$$

$$\Rightarrow \vec{F} = \frac{F}{2}(\sqrt{3}\hat{i} - \hat{k}) = \frac{2mv}{\sqrt{3}\Delta t}(\sqrt{3}\hat{i} - \hat{k})$$

Equilibrium of force in Z-direction (acting on wedge)

$$F_y + mg = N$$

$$\Rightarrow N = \frac{F}{2} + mg = \frac{2mv}{\sqrt{3}\Delta t} + mg$$

$$N = \left(\frac{2mv}{\sqrt{3}\Delta t} + mg\right)\hat{k}$$

This is the normal force exerted by the table on the wedge during the time Δ t.

(b) Torques on wedge about the centre of mass of the wedge

$$F \times h - \text{Torque due to } N + mg \times 0 = 0$$

Here h is the perpendicular distance between the centre of mass of the wedge and the line of action of F

$$\therefore\ \text{Torque due to } N = F \times h = \frac{4mv}{\sqrt{3}\Delta t} \times h$$

37. The disc-rod system can roll on the truck without slipping as the friction of the disc-rod system with the floor of the moving truck is large.

Given : Mass of each disc m = 2 kg

Radius of each disc = R = 10 cm = 0.1 m

Length of rod = 20 cm = 0.2 m

Acceleration of truck along x-axis = 9 m/s^2

The axis of disc-rod object is horizontal and perpendicular to the direction of the motion of truck. z-axis is the vertically upward direction.

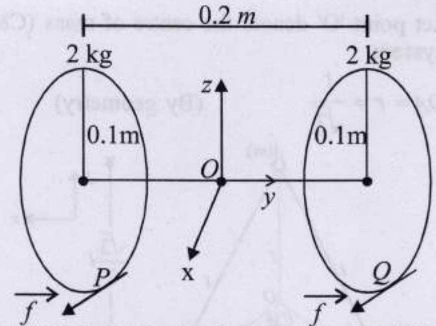
(i) Let a_0 = linear acceleration of centre of mass of disc

α = angular acceleration about its centre of mass

$$\therefore\ a_0 = \frac{f}{m} = \frac{f}{2} \quad \text{...(i)}$$

$$\text{Also}\quad \alpha = \frac{\tau}{I} = \frac{fR}{mR^2/2} = \frac{2f}{mR} = \frac{2f}{2 \times 0.1} = 10f$$

$$\therefore\ \alpha = 10f \quad \text{...(ii)}$$



Since there is no slipping between disc and truck, acceleration of P = acceleration of Q

$$a_0 + R\alpha = a$$

or $\left(\frac{f}{2}\right) + (0.1)(10f) = a$ (from (i))

or $\frac{3f}{2} = a$ or $f = \frac{2a}{3}$ or $f = \frac{2 \times 9.0}{3} = 6N$

This force of friction f, is acting in positive x-direction. $\hat{i}$ is unit vector along x-axis.

$\therefore$ In vector form, $\vec{f} = (6\hat{i})$ newton ... (iii)

(ii) Frictional torque, $\tau = r \times f$

At P, $\vec{r}_1 = -0.1\hat{j} - 0.1\hat{k}$

At Q, $\vec{r}_2 = 0.1\hat{j} - 0.1\hat{k}$ $\therefore\ \vec{\tau}_1 = \vec{r}_1 \times \vec{f}$

or $\vec{\tau}_1 = \left(-0.1\hat{j} - 0.1\hat{k}\right) \times \left(6\hat{i}\right) N\text{-}m$

or $\vec{\tau}_1 = 0.6\hat{k} - 0.6\hat{j}$

or $\vec{\tau}_1 = 0.6(\hat{k} - \hat{j}) N\text{-}m$ ... (iv)

$$|\vec{\tau}_1| = \sqrt{(0.6)^2 + (0.6)^2} = \sqrt{2} \times 0.6 = 1.414 \times 0.6$$

$$= 0.85\ \text{N-m}$$

Similarly, $\vec{\tau}_2 = \vec{r}_2 \times \vec{f}$ or $\tau_2 = 0.6\left(-\hat{j} - \hat{k}\right)$ N-m

$$|\vec{\tau}_2| = \sqrt{(0.6)^2 + (0.6)^2} = \sqrt{2} \times 0.6 = 0.85\ \text{N-m}$$

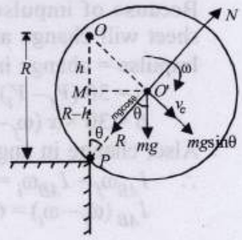
$$|\vec{\tau}_1| = |\vec{\tau}_2| = 0.85\ \text{N-m}$$

38. (a) The cylinder of radius R mass 'm' rolls off the edge of the rectangular block without slipping. The rotation of the cylinder is about the point of contact so energy of the cylinder is conserved.

Let the original position of centre of mass of the cylinder be O. While rolling down off the edge, let the cylinder be at such a position that its centre of mass is at a position O'.

Let $\angle NPO$ be θ. As the cylinder is rolling, the c.m. rotates in a circular path. The centripetal force required for the circular motion.

$$mg\cos\theta - N = \frac{mv_c^2}{R}$$



The condition for the cylinder leaving the edge, $N = 0$

$\therefore \quad mg\cos\theta = \dfrac{mv_c^2}{R} \Rightarrow \cos\theta = \dfrac{v_c^2}{Rg}$ ... (i)

Applying energy conservation from O to O'.
Loss of potential energy of cylinder
= gain in translational $K.E.$ + gain in rotational $K.E.$

$$mgh = \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2 \quad \text{... (ii)}$$

Where I is the moment of inertia of the cylinder about O', $I = \frac{1}{2}mR^2$, ω is the angular speed, V_c is the velocity of center of mass.

Also for rolling, $v_c = \omega R \Rightarrow \omega = \dfrac{v_c}{R}$

Putting value of I and ω in eqn. (ii)

$$mgh = \frac{1}{2}mv_c^2 + \frac{1}{2}\times\frac{1}{2}mR^2 \times \frac{v_c^2}{R^2}$$

$$\Rightarrow \quad gh = \frac{1}{2}v_c^2 + \frac{1}{4}v_c^2 = \frac{3}{4}v_c^2 \Rightarrow v_c^2 = \frac{4gh}{3}$$

In $\Delta O'MP$, $\cos\theta = \dfrac{R-h}{R} \Rightarrow h = R(1-\cos\theta)$

$$\therefore \quad v_c^2 = \frac{4g}{3}R(1-\cos\theta) \quad \text{... (iii)}$$

From eq. (i) and (iii), we get

$$\cos\theta = \frac{4gr}{3Rg}(1-\cos\theta)$$

$$\Rightarrow 3\cos\theta = 4 - 4\cos\theta \Rightarrow \cos\theta = \frac{4}{7}$$

(b) Speed of C.M. of cylinder before leaving contact with edge.

$$v_c^2 = \frac{4gR}{3}\left(1-\frac{4}{7}\right) = \frac{4gR}{7} \Rightarrow v_c = \sqrt{\frac{4gR}{7}} \quad \text{[from (iii)]}$$

(c) Before the centre of mass of the cylinder reaches the horizontal line of the edge, it leaves contact with the edge

$$\theta = \cos^{-1}\frac{4}{7} = 55.15°$$

Hence the rotational $K.E.$, which the cylinder gains at the time of leaving contact with the edge remains the same in its further motion. Thereafter the cylinder gains translational $K.E.$ Again applying energy conservation from O to the point where centre of mass is in horizontal line with edge

$$mgR = \frac{1}{2}I\omega^2 + \frac{1}{2}m(v'_c)^2$$

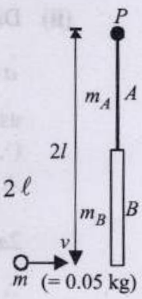
$$mgR = \frac{1}{2}\times\frac{1}{2}mR^2 \times \left(\sqrt{\frac{4g}{7R}}\right)^2 + \frac{1}{2}m(v'_c)^2$$

$$\because \quad \omega = \frac{v_c}{R} = \sqrt{\frac{4gR/7}{R}}$$

$$\Rightarrow \quad mgR - \frac{mgR}{7} = \frac{6mgR}{7} = \text{translational } K.E.$$

Also, rotational $K.E. = \dfrac{1}{2}I\omega^2 = \dfrac{mgR}{7}$

$$\therefore \quad \frac{\text{Translational } K.E.}{\text{Rotational } K.E.} = 6$$



39. Since, $\tau = \dfrac{dL}{dt}$ and $\tau = 0 \quad \therefore L$ is constant.

Angular momentum before collision = $mv \times 2\ell$
Angular momentum after collision = $I\omega$

$$I = I_m + I_A + I_B$$

$$= \left[m(2\ell)^2 + \left\{m_A\left(\frac{\ell^2}{12}\right) + \left(\frac{\ell}{2}\right)^2\right\} + \left\{m_B\left(\frac{\ell^2}{12}\right) + \left(\frac{\ell}{2}+\ell\right)^2\right\}\right]$$

$$= \left[4m\ell^2 + m_A\left(\frac{\ell^2}{12}+\frac{\ell^2}{4}\right) + m_B\left(\frac{\ell^2}{12}+\frac{9\ell^2}{4}\right)\right]$$

$$= \left[4m\ell^2 + \frac{1}{3}m_A\ell^2 + \frac{7}{3}m_B\ell^2\right] = 0.09\,\text{kg } m^2$$

$$I\omega = mv \times 2\ell$$

$$\Rightarrow \quad \omega = \frac{mv\times 2\ell}{I} = \frac{0.05\times v\times 2\times 0.6}{0.09} = 0.67\,v$$

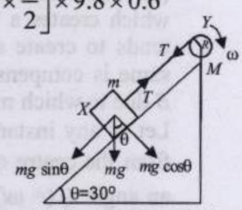
Applying conservation of mechanical energy after collision.
Loss of $K.E.$ = gain in $P.E.$

$$\frac{1}{2}I\omega^2 = mg(2\ell) + m_A\left(\frac{\ell}{2}\right)g + m_B g\left(\frac{3\ell}{2}\right)$$

$$\Rightarrow \quad \frac{1}{2}\times 0.09\times(0.67v)^2$$

$$= \left[0.05\times 2 + 0.01\times\frac{1}{2} + 0.02\times\frac{3}{2}\right]\times 9.8\times 0.6$$

$$\Rightarrow \quad v = 6.3 \text{ m/s}$$



40. **(i)** According to question, the drum is given an initial angular velocity such that the block X starts moving up the plane.

As the time passes and block X rises up the velocity of the block decreases. Let a be the linear retardation of the block X.

$\therefore \quad mg\sin\theta - T = ma$ ... (i)

The linear retardation of the block a and the angular acceleration of the drum (α) are related as

$$a = R\alpha \Rightarrow \alpha = \frac{a}{R}$$

where R is the radius of the drum.
The retarding torque of the drum is due to tension T in the string.

$$\tau = T\times R = I\alpha \; (\because \tau = I\alpha)$$

$$\therefore \quad T\times R = \frac{1}{2}MR^2\alpha \qquad \left[\because I = \frac{1}{2}MR^2\right]$$

$$\Rightarrow TR = \frac{1}{2}MR^2\frac{a}{R} \Rightarrow a = \frac{2T}{M}$$

Substituting this value of a in eq. (i)

$$mg\sin\theta - T = m\times\frac{2T}{M} \Rightarrow mg\sin\theta = \left(1+\frac{2m}{M}\right)T$$

$$\therefore \quad T = \frac{(mg\sin\theta)\times M}{M+2m} = \frac{0.5\times 9.8\times\sin 30°\times 2}{2+2\times 0.5} = 1.63\text{ N}$$

(ii) Distance travelled by x

$$a = \frac{2T}{M} = \frac{2\times1.63}{2} = 1.63\ m/s^2$$

using $V = R\omega \Rightarrow V = 0.2\times10 = 2$ m/s

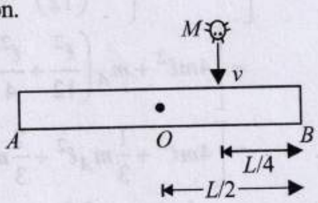
($\because$ $R = 0.2\ m$ and $\omega = 10$ rod/s given)

$\therefore$ Distance travelled by block x

$$2as = v^2 - u^2 \Rightarrow s = \frac{v^2}{2a} = \frac{2\times2}{2\times1.63}$$

$\therefore$ Distance travelled by $x = 1.22$ m

41. **(a)** Angular momentum of the system (rod + insect) about the centre of mass 'O' before collision = angular momentum of the system after collision.



$$Mv\times\frac{L}{4} = I\omega$$

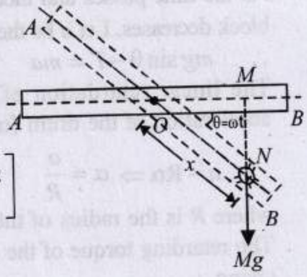
Where I is the moment of inertia of the system just after collision and ω is the angular velocity just after collision.

$$\Rightarrow\ Mv\frac{L}{4} = \left[M\left(\frac{L}{4}\right)^2 + \frac{1}{12}ML^2\right]\omega$$

$$\Rightarrow\ Mv\times\frac{L}{4} = \frac{ML^2}{4}\left[\frac{1}{4}+\frac{1}{3}\right]\omega = \frac{ML^2}{4}\left[\frac{3+4}{12}\right]$$

$$= \frac{ML^2}{4}\times\frac{7}{12}\times\omega \Rightarrow \omega = \frac{12}{7}\frac{v}{L}.$$

(b) After collision there is an extra mass M of the insect which creates a torque in the clockwise direction, which tends to create angular acceleration in the rod. But the same is compensated by the movement of insect towards B due to which moment of inertia I of the system increases. Let at any instant of time t the insect be at a distance x from the centre of the rod and the rod has turned through an angle θ (= ωt) w.r.t its original position



$$\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega)$$

$$= \omega\frac{dI}{dt}$$

$$= \omega\frac{d}{dt}\left[\frac{1}{12}ML^2 + Mx^2\right]$$

$$= 2M\omega x\frac{dx}{dt}$$

This torque is balanced by the torque due to weight of insect.

$$\therefore\ 2M\omega x\frac{dx}{dt} = Mg\,(x\cos\theta)\ \Rightarrow dx = \left(\frac{g}{2\omega}\right)\cos\omega t\,dt$$

On integration, taking limits

$$\int_{L/4}^{L/2} dx = \frac{g}{2\omega}\int_0^{\pi/2\omega}\cos\omega t\,dt$$

when $x = \frac{L}{4}, \omega t = 0$

$$[x]_{L/4}^{L/2} = \frac{g}{2\omega^2}[\sin\omega t]_0^{\pi/2\omega}$$

when $x = \frac{L}{2}, \omega t = \frac{\pi}{2}$

$$\Rightarrow\ \left(\frac{L}{2}-\frac{L}{4}\right) = \frac{g}{2\omega^2}\left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$\Rightarrow\ \frac{L}{4} = \frac{g}{2\omega^2} \Rightarrow \omega = \sqrt{\frac{2g}{L}}$$

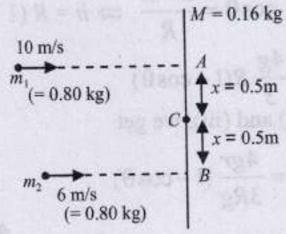
But $\omega = \frac{12}{7}\frac{v}{L} \Rightarrow \frac{12}{7}\frac{v}{L} = \sqrt{\frac{2g}{L}} \Rightarrow v = \frac{7}{12}\sqrt{2gL}$

$$\Rightarrow\ v = \frac{7}{12}\sqrt{2\times10\times1.8} = 3.5\ \text{ms}^{-1}$$

42. Before collision, Kinetic energy of the system

$$\text{K.E.}_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{2}0.08\times10^2 + \frac{1}{2}0.08\times6^2 + 0 = 5.44\ \text{J} \quad \text{... (i)}$$



Applying law of conservation of linear momentum during collision

$$m_1\times v_1 + m_2\times v_2 = (M+m_1+m_2)\,V_c$$

where V_c is the velocity of centre of mass of the bar and particles sticked on it after collision

$0.08\times10 + 0.08\times6 = (0.16+0.08+0.08)\,V_c \Rightarrow V_c = 4$ m/s

$\therefore$ K.E. trans after collision

$$= \frac{1}{2}(M+m_1+m_2)V_c^2 = 2.56\ \text{J} \quad \text{...(ii)}$$

Applying conservation of angular momentum of the bar and two particle system about the centre of the bar.

$T_{External} = 0 \therefore L$ constant

$L_i = m_1v_1\times x - m_2v_2x$

$= 0.08\times10\times0.5 - 0.08\times6\times0.5$

$= 0.4 - 0.24 = 0.16$ kg m^{2}s^{-1} (In clockwise direction)

$L_f = I\omega$

$$= \left[\frac{M\ell^2}{12} + m_1x^2 + m_2x^2\right]\omega$$

$$= \left[\frac{(0.16)(\sqrt{3})^2}{12} + 0.08\times(0.5)^2 + (0.08)(0.5)^2\right]\omega$$

$= 0.08\ \omega$

$\therefore$ $0.08\ \omega = 0.16 \Rightarrow \omega = 2$ rad/s ... (iii)

$$\text{K.E.}_{\cdot\text{Rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\times0.08\times2^2 = 0.16\ \text{J} \quad \text{... (iv)}$$

$K.E._f$ = Translational $K.E.$ + Rotational $K.E.$
$= 2.56 + 0.16 = 2.72$ J
$\Delta K.E. = K.E._i$ – Final $K.E._f$
$= 5.44 - 2.72 = 2.72$ J

43. $P.E.$ at $D = P.E.$ at $Q + (K.E.)_{Trans} + (K.E.)_{Rot}$
From energy conservation principle

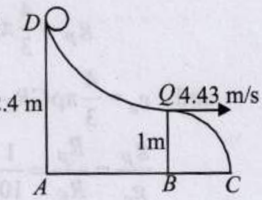
$$\Rightarrow\ mg(2.4) = mg(1) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mg(2.4-1) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2}$$

$$\left(\because\ I = \frac{2}{5}mr^2\ \&\ \omega = \frac{v}{r}\right)$$

or, $g \times 1.4 = \dfrac{7v^2}{10} \Rightarrow v = 4.43$ m/s

After point Q, the path of the body parabolic.



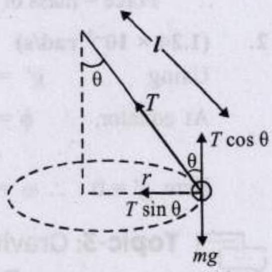
$h = \dfrac{1}{2}gt^2 \Rightarrow 1 = 4.9\,t_y^2$

$\therefore\ t_y = \dfrac{1}{\sqrt{4.9}} = 0.45$ sec

Distances $BC = V \times t_y = 4.43 \times 0.45 = 2$m
During its flight as a projectile, the sphere continues to rotate because of conservation of angular momentum.

44. Let small mass m moves around a circular path of radius r. Let the string makes an angle θ with the vertical and T be the tension

$T\cos\theta = mg$ ... (i)
$T\sin\theta = mr\omega^2$ ... (ii)



$\therefore\ \tan\theta = \dfrac{r\omega^2}{g}$

From figure, $\sin\theta = \dfrac{r}{\ell}$

$\Rightarrow\ r = \ell\sin\theta$

$\therefore\ \tan\theta = \ell\sin\theta\dfrac{\omega^2}{g}$

$\Rightarrow\ \omega^2 = \dfrac{\tan\theta . g}{\ell\sin\theta}$ or, $\omega = \sqrt{\dfrac{g}{\ell\cos\theta}}$

$\Rightarrow v = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{\ell\cos\theta}}\ \because\ \omega = 2\pi v$ ... (iii)

For M to remain stationary, $T = Mg$
and $T\cos\theta = mg$

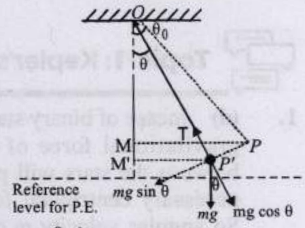
$\therefore\ Mg\cos\theta = mg$

$\Rightarrow\ \cos\theta = \dfrac{m}{M}$

Putting this value of cos θ is eq (iii), we get

$$v = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}\frac{M}{m}}$$

45. In ΔOPM, $\cos\theta_0 = \dfrac{OM}{\ell}$



$\Rightarrow OM = \ell\cos\theta_0$

In $\Delta OP'M'$, $\cos\theta = \dfrac{OM'}{\ell}$

$\Rightarrow OM' = \ell\cos\theta$

$OM' - OM = \ell(\cos\theta - \cos\theta_0)$

Loss in potential energy = Gain in kinetic energy
(Activity P to P')

$\Rightarrow\ mg\ell(\cos\theta - \cos\theta_0) = \dfrac{1}{2}mv^2$

$\Rightarrow\ v^2 = 2g\ell(\cos\theta - \cos\theta_0)$ ... (i)

$T - mg\cos\theta = \dfrac{mv^2}{\ell}$

$\therefore\ T = \dfrac{mv^2}{\ell} + mg\cos\theta$ ... (ii)

From eg. (i) and (ii)

$T = \dfrac{m}{\ell} \times 2g\,\ell(\cos\theta - \cos\theta_0) + mg\cos\theta$

$\therefore\ T = 3mg\cos\theta - 2\,mg\cos\theta_0$

$\Rightarrow\ T = mg\,(3\cos\theta - 2\cos\theta_0)$

From equation (ii) it is clear that the tension is maximum when $\cos\theta = 1$ i.e., $\theta = 0°$

Hence, $T_{max} = \dfrac{mv^2}{\ell} + mg$ ... (iii)

From eqn. (i)

$v^2 = 2g\ell(1 - \cos\theta_0)$ ... (iv)

From (iii) and (iv)

$T_{max} = \dfrac{m}{\ell}[2g\ell(1 - \cos\theta_0)] + mg$

$\therefore\ T_{max} = 3mg - 2mg\cos\theta_0$

$80 = 3 \times 40 - 2 \times 40\cos\theta_0$

$\Rightarrow\ 80\cos\theta_0 = 40 \Rightarrow \cos\theta_0 = \dfrac{1}{2} \Rightarrow \theta_0 = 30°$